When $u = 1$, $\theta = \theta'$ and

$$E_{\theta'}(n) = \frac{ab}{E_{\theta'}[A(x) + cB(x)]^2}. \tag{2.18}$$

The author wishes to express his thanks to L. J. Savage for his interest in this problem and his many useful suggestions.

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WHEN DIFFERENT PAIRS OF HYPOTHESES HAVE THE SAME FAMILY OF LIKELIHOOD-RATIO TEST REGIONS[1]

By Leonard J. Savage

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Blasbalg [1], in this issue of these *Annals*, shows that certain families of distributions are especially simple, or degenerate, from the point of view of sequential tests. The main object of this note is to show briefly that these are (at least practically) the only families thus degenerate; some preliminary and related conclusions are also demonstrated.

Let F and G be a pair of probability measures on a space X with elements x, and let ℓ be the logarithm of the likelihood ratio of F with respect to G. ℓ is of course defined only mod $(F + G)$, that is, only up to sets simultaneously of F and G measure 0. If x_i is a sequence of values of x, then a likelihood-ratio critical region in X^n is defined by

$$R(A, n) = \left\{ (x_1, \cdots, x_n) : \sum_1^n \ell(x_i) \leqq A \right\}. \tag{1}$$

The innocuous ambiguity of ℓ of course induces corresponding ambiguity in R.

This family of sets R is simplest to study when the distribution of ℓ is non-

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