

Attention should be drawn to a paper of Korolyuk [2] wherein the author gives different versions of the probabilities we have presented for the case  $x = y$ .

REFERENCES

- [1] J. BLACKMAN, "An extension of the Kolmogorov distribution," *Ann. Math. Stat.*, Vol. 27 (1956), pp. 513-520.
- [2] V. S. KOROLYUK, "On the difference of the empirical distribution of two independent samples," *Izv. Akad. Nauk. SSSR*, Vol. 19 (1955), pp. 81-96.

APPENDIX

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By a *path* of length  $n$  we shall mean an ordered sequence of  $n + 1$  integers  $(z_0, \dots, z_n)$ , such that

$$z_i - z_{i-1} \geq -1 \quad (i = 1, \dots, n).$$

For each path  $\pi_n = (z_0, \dots, z_n)$ , let

$$P(\pi_n) = \prod_{i=1}^n p(z_i - z_{i-1}),$$

(the weight or "probability" of  $\pi_n$ ). Here, the  $p_i = p(i)$ , ( $i = -1, 0, +1, \dots$ ), denote given (real or complex) numbers,  $p(-1) \neq 0$ . Finally, let

$$e_x(n) = \sum_{\pi_n}' p(\pi_n),$$

the summation being extended over all the paths  $\pi_n = (z_0, \dots, z_n)$  with  $z_0 = 0$ ,  $z_n = z$ ,  $z_i \neq z$  ( $i = 0, 1, \dots, n - 1$ ).

THEOREM. For  $n = 1, 2, \dots$ ,

$$(8) \quad e_x(n) = -zr_x(n)/n + \sum_{j=1}^{\infty} j(j+1)p_j \sum_{0 < m \leq +z} r_x(-m)r_{-j}(m+n-1)/(m+n-1).$$

Here, for arbitrary integers  $h$  and  $s$ ,  $r_h(s)$  is defined as the coefficient of  $w^{h+s}$  in the formal development

$$(p_{-1} + p_0w + p_1w^2 + \dots)^s = \sum_h r_h(s)w^{h+s};$$

especially,  $r_h(s) = 0$  if  $h + s < 0$ .

PROOF. Let  $n$  and  $z$  be given integers,  $n \geq 1$ . For any path  $(z_0, \dots, z_n)$  with  $z_0 = 0$ ,  $z_n = z$ , we have

$$z_i - z_{i-1} = z - \sum_{\substack{\nu=1 \\ \nu \neq i}}^n (z_\nu - z_{\nu-1}) \leq z + n - 1,$$

