ANOTHER COUNTABLE MARKOV PROCESS WITH ONLY INSTANTANEOUS STATES

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Let P be the transition function for a Markov process with a countable state space A and stationary transition probabilities; i.e., P is a nonnegative function defined for all triples (a, b, t) with $a \varepsilon A$, $b \varepsilon A$, and t a nonnegative real number, satisfying

$$(1) \qquad P(a, b, 0) = 1 \text{ if } a = b, \qquad 0 \text{ if } a \neq b,$$

$$(2) \qquad \sum_b P(a, b, t) = 1 \quad \text{for all} \quad a, t,$$

and

$$(3) \qquad P(a, b, s + t) = \sum_{c \varepsilon A} P(a, c, s)P(c, b, t) \quad \text{for all} \quad s \geqq 0, \qquad t \geqq 0, a, b.$$

We shall suppose, as usual, that P is continuous at $t = 0$; i.e.,

$$(4) \qquad P(a, a, t) \to 1 \text{ as } t \to 0 \text{ for all } a.$$

It is well known that, for any P satisfying (1), (2), (3), and (4), $P'(a, a, 0)$ exists for all a (it may be negatively infinite). Following P. Lévy [2], a state is called "instantaneous" if $P'(a, a, 0) = -\infty$. Examples of processes with all states instantaneous have been given by Feller and McKean [2] and by Dobrushin [1]. The purpose of this note is to describe a third example, somewhat simpler than those previously given.

We first describe the process informally, after which we define P and verify (1), (2), (3), and (4) and $P'(a, a, 0) = -\infty$ for all a directly. Let $X_1(t)$, $X_2(t)$, $\cdots$ be a sequence of Markov processes, independent of each other, each with two states 0 and 1. We suppose $X_n(0) = 0$ for all n. Let $X_n(t)$ be characterized by the parameters λ_n, μ_n:

$$\Pr\{X_n(t + h) = 1 \mid X_n(t) = 0\} = \lambda_n h + o(h),$$

$$\Pr\{X_n(t + h) = 0 \mid X_n(t) = 1\} = \mu_n h + o(h).$$

Our process $X(t)$ will be the joint process $X_1(t)$, $X_2(t)$, $\cdots$ which is clearly a Markov process. To insure that $X(t)$ has only a countable set of states, we

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