A CONSTRUCTION FOR ROOM'S SQUARES AND AN APPLICATION IN EXPERIMENTAL DESIGN

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1. T. G. Room [1] recently proposed the following problem: To arrange the $n(2n - 1)$ symbols rs (which is the same as sr) formed from all pairs of $2n$ different digits in a square of $2n - 1$ rows and columns such that in each row and column there appear n symbols (and $n - 1$ blanks) which among them contain all $2n$ digits.

He remarked that the problem is soluble when $n = 1$ (trivially) and $n = 4$ but not when $n = 2$ or 3; and he gave one solution for $n = 4$.

Squares of such a type have uses in experimental designs. We explain below a simple construction for squares where n has the form 2^{2m-1}. Each square constructed in this way is represented in a canonical form by applying a well-known theorem of J. Singer [2]. In this form as soon as the top row of entries in a square is known, all the other entries may be written down immediately by means of a straight-forward cyclic process. Thus an index of first rows is all that is necessary to catalogue squares in their canonical forms.

It may be permissible to give here a slight modification of the proof of Singer's theorem in order to show a natural application of the regular representation of linear algebras.

2. Let $\mathfrak{A}$ be a linear associative algebra, of order m and with modulus, over a commutative field K. It is well known that $\mathfrak{A}$ is isomorphic with an algebra of $m \times m$ matrices whose elements belong to K (c.f. Macduffee [3], Section 123).

A Galois field $GF(p^{mn})$ is such a linear algebra over a $GF(p^n)$. If the elements of the $GF(p^{mn})$ are $0, \alpha, \alpha^2, \cdots, \alpha^{p^{mn}-1} = 1$ the irreducible equation, of degree m and with coefficients in $GF(p^n)$,

$$f(x) \equiv x^m - a_1x^{m-1} - \cdots - a_m = 0$$

which is satisfied by α is called primitive (Dickson [4], Section 35). A basis for the algebra consists of $1, \alpha, \alpha^2, \cdots, \alpha^{m-1}$ and the modulus is 1.

The primitive equation is both the minimum and characteristic equation of the companion matrix

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ 0 & 0 & 0 & \cdots & 1 \\ a_m & a_{m-1} & a_{m-2} & \cdots & a_1 \end{pmatrix}$$

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