NOTE ON SUFFICIENT STATISTICS AND TWO-STAGE PROCEDURES

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0. Introduction. This note is the result of an attempt to discover problems in which one can apply the two-stage procedure used by Stein [1] for tests regarding the mean of a normal population. One such problem, that of testing for a location parameter of an exponential population, was found to be easily soluble along the lines of Stein's work. An investigation of the problem of optimum statistics for such procedures was also undertaken, and partial solutions, given in Sec. 2, were found. In this connection, the author would like to thank the referee for his useful comments.

1. Testing for a location parameter of a distribution. Throughout this paper $F(x)$ will be a one-dimensional c.d.f. with at least two points of increase. Further, $\{Y_n\}$ will always denote a sequence of independent random variables having a common c.d.f. $F(x)$ and $\{X_n\}$ will denote a family of sequences of independent random variables, all elements of any one sequence having a common c.d.f. $F[(x - \theta)/\sigma]$, $-\infty < \theta < \infty, \sigma > 0$. We shall be dealing with statistics or sequences of real and single-valued functions $t(n; x_1, \cdots, x_n)$ and $s(n; x_1, \cdots, x_n)$ of n real variables, $n = 1, 2, \cdots$, about which one or more of the following assumptions will be made as required:

Assumption I. For any integer $n > 0$, any $a > 0$, any real b and any

$$(x_1, \cdots, x_n) \,\varepsilon\, R^n,$$

$$t(n; ax_1 + b, \cdots, ax_n + b) = at(n; x_1, \cdots, x_n) + b. \tag{1}$$

Assumption II. Analogously,

$$s(n; ax_1 + b, \cdots, ax_n + b) = as(n; x_1, \cdots, x_n). \tag{2}$$

Assumption III. There exists a positive, nondecreasing and unbounded sequence $k(n)$ such that

$$\Pr \{t(n; Y_1, \cdots, Y_n) \leqq x/k(n)\} = G(x) \tag{3}$$

is independent of n. Without loss of generality, we may assume $k(1) = 1$.

Assumption IV. The random variables $t(n; Y_1, \cdots, Y_n)$ and $s(n; Y_1, \cdots, Y)_n$ are stochastically independent.

Assumption V. There exists a positive integer m, such that for any $n > m$, $t(n; x_1, \cdots, x_n)$ is a function only of $m, n, t(m; x_1, \cdots, x_m)$ and $x_{m+1}, \cdots, x_n$.

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