PÓLYA TYPE DISTRIBUTIONS IV. SOME PRINCIPLES OF SELECTING A SINGLE PROCEDURE FROM A COMPLETE CLASS[1]

By Samuel Karlin

Stanford University

0. Introduction. In previous publications [1], [2], and [3], various aspects of decision theory in which the underlying distributions are Pólya type have been studied. For example, complete classes of decision procedures were determined, all Bayes procedures were characterized, and the problem of admissibility was investigated as related to various kinds of loss functions.

Usually the minimal complete class of decision procedures, to which the statistician would obviously restrict himself in practical application, is still quite large. Consequently, without any additional knowledge or further conditions, it is a hopeless task to justify preferring any given admissible procedure to another. It is therefore of importance to introduce new criteria which will single out a procedure for use. It is the object of this paper to discuss some further principles which select a single statistical procedure from the class of all "monotone" procedures.

In the $n = 2$ action problem (essentially the testing problem) some of the classical principles used to determine a single admissible procedure for use are related to the concepts of unbiasedness, maximum likelihood, invariance, minimax, etc. These principles have received much attention and their justification and relevance are well understood for the parametric testing problem. For a detailed analysis of these classical concepts in the case of two action problems when the underlying distributions are Pólya type, the reader is referred to [1]. Our present discussion deals with the extension and analysis of some of these principles to the n-action problem. In the sense that the estimation problem may be obtained as a limit of finite action problems, the ideas here shed further light on the estimation problem.

The language and notation we use is that of the introduction of the previous paper [3]. However, a knowledge of the results of [3] is not necessary for an understanding of the present discussion although a reading of the introduction would more than provide sufficient familiarity with the terminology to be used here as well as a general background for Pólya type distributions. Henceforth, we assume that the notation of this manuscript is that of [3]. Nevertheless, for clarity of exposition, we review briefly some of the main quantities to be used.

Let the distribution of the observed real random variable X (usually a sufficient

Received March 25, 1957.

[1] Research sponsored by the Office of the Naval Research Nonr 225 (21) (NR 042-993). Research on the paper "Pólya-Type Distributions III: Admissibility of Multi-Action Problems," in the December, 1957 issue of these *Annals*, was done under the same contract.