ABSTRACTS OF PAPERS

(Abstracts of papers presented at the Ames, Iowa Meeting of the Institute, April 3–5, 1958.)

11. **Bias and Confidence in Not-quite Large Samples.** (Preliminary Report) JOHN W. TUKEY, Princeton University, (By Title).

The linear combination of estimates based on all the data with estimates based on parts thereof seems to have been first treated in print as a means of reducing bias by Jones (*J. Amer. Stat. Assn.*, Vol. 51 (1956), pp. 54–83). Let $y_{(.)}$ be the estimate based on all the data, $y_{(i)}$ that based on all but the ith piece, $\bar{y}_{(i)}$ the average of the $y_{(i)}$. Quenouille (*Biometrika*, Vol. 43 (1956), pp. 353–560) has pointed out some of the advantages of $ny_{(.)} - (n-1)\bar{y}_{(i)}$ as such an estimate of much reduced bias. Actually, the individual expressions $ny_{(.)} - (n-1)y_{(i)}$ may, to a good approximation, be treated as though they were n independent estimates. Not only is each nearly unbiased, but their average sum of squares of deviations is nearly $n(n-1)$ times the variance of their mean, etc. In a wide class of situations they behave rather like projections from a non-linear situation on to a tangent linear situation. They may thus be used in connection with standard confidence procedures to set closely approximate confidence limits on the estimand. (Received December 26, 1957.)

12. **Limiting Distributions of k-sample Test Criteria of Kolmogorov-Smirnov-v. Mises Type.** J. KIEFER, Cornell University, (By Title).

Let S_j be the sample d.f. of n_j independent, indentically distributed random variables with common unknown continuous d.f. F_j $(1 \leqq j \leqq k)$, the S_j being independent. For testing the hypothesis $H:F_1 = \cdots = F_k$, several criteria were suggested by the author in *Ann. Math. Stat.*, Vol. 26 (1955), p. 775. Among these are $T = \sup_x \sum_j n_j[S_j(x) - \bar{S}(x)]^2$ and

$$W = \int \textstyle\sum_j n_j[S_j(x) - \bar{S}(x)]^2 dS^*(x),$$

where $\bar{S} = \sum_j n_j S_j / \sum n_j$ and $S^* = \sum_j a_j S_j$ with $\sum_j a_j = 1$. It is proved by the method indicated in the above reference that, under H, the limit of $P\{T < a^2\}$ as all $n_j \to \infty$, is

$$\frac{2^{(5-2k)/2}a^{1-k}}{\Gamma((k-1)/2)} \sum_{n=1}^{\infty} \frac{\alpha_n^{k-3} \exp[-\alpha_n^2/2a^2]}{[J_{(k-1)/2}(\alpha_n)]^2},$$

where $a > 0$ and α_n is the nth positive zero of the Bessel function $J_{(k-3)/2}$; alternative expressions are also given. When $k = 4$ or $k = 2$ the summand above reduces to an elementary function; the latter case gives the Kolmogorov-Smirnov distribution, since $T^{1/2}$ is the Smirnov statistic when $k = 2$. The limiting d.f. of W is expressible in a series involving Hermite polynomials when k is odd and Bessel functions when k is even. For $k = 2$, W is the test suggested by Lehmann and Rosenblatt, and the above d.f. is the limiting ω^2 d.f. in the form given by Anderson and Darling. (Received January 6, 1958.)

13. **A Rule for Action Based on Percentage Changes in the Sample Mean.** D. B. OWEN, Sandia Corporation, (By Title).

A random selection is made of n items from a normal population X, each item is measured once, and the sample mean $\bar{x}$ is computed. The sample items are identified by some means and the sample and the remaining population are mixed at random. They are then subjected to some condition, such as storage, after which the same items that were first sam-