$$\varphi_2(t) = \frac{1}{\cosh 2t} + i\sqrt{2}\,\frac{\sinh t}{\cosh 2t},$$

$$\varphi_3(t) = \frac{1}{(\cosh 2t)^{\frac{1}{2}}} \quad \text{and} \quad \varphi_4(t) = \frac{e^{it}}{(\cosh 2t)^{\frac{1}{2}}}.$$

If these functions φ are inverted (see [1], pp. 388–389, and [2], p. 30) and a change of variable made from $(4/\pi) \log | X |$ to X, assuming X symmetric, then the corresponding density functions are

$$p_1(x) = \frac{\sqrt{2}}{\pi}\,\frac{x^2}{1 + x^4}, \qquad -\infty < x < +\infty,$$

$$p_2(x) = \frac{2}{\pi}\,\frac{x^4}{(1 + x^2)(1 + x^4)}, \qquad -\infty < x < +\infty,$$

$$p_3(x) = \frac{1}{2\pi^2\sqrt{2\pi}\,|x|}\left|\Gamma\left(\frac{1}{4} + \frac{i \log |x|}{\pi}\right)\right|^2, \qquad -\infty < x < +\infty,$$

and

$$p_4(x) = \frac{1}{2\pi^2\sqrt{2\pi}\,|x|}\left|\Gamma\left(\frac{1 - i}{4} + \frac{i \log |x|}{\pi}\right)\right|^2, \qquad -\infty < x < +\infty.$$

Using $\theta(-t)$ instead of $\theta(t)$ provides additional densities $p^*(x) = p(1/x)/x^2$ (if p is the density function of X then p^* is the density function of $1/X$). For example,

$$p_1^*(x) = \frac{\sqrt{2}}{\pi}\,\frac{1}{1 + x^4}, \qquad -\infty < x < +\infty,$$

and

$$p_2^*(x) = \frac{2}{\pi}\,\frac{1}{(1 + x^2)(1 + x^4)}, \qquad -\infty < x < +\infty.$$

REFERENCES

[1] J. D. Bierens de Haan, *Nouvelles tables d'Intégrals Définies*, G. E. Stechert, 1939.
[2] A. Erdélyi, W. Magnus, F. Oberhettinger, and F. G. Tricomi, *Tables of integral transforms*, Vol. 1, McGraw-Hill, 1954.

ESTIMATION OF A REGRESSION LINE WITH BOTH VARIABLES SUBJECT TO ERROR UNDER AN UNUSUAL IDENTIFICATION CONDITION

By Herman Rubin

University of Oregon

Suppose the random variables $w_j = (\xi_j, u_j, v_j)$ are independently and identically distributed with joint distribution F. Then if $\int\int\int e^{\alpha u + \beta v}\, dF(\xi, u, v)$ exists

Received December 13, 1956; revised February 10, 1958.