ticular, if cov $\{y_k - t, t\} < 0$, a value of $\lambda > 0$ will provide lesser variance than for $\lambda = 0$.

From the result it is easy to show that:

(D) *If $F(q)$ is subexponential to the right, then no single order statistic, except possibly the righthandmost, is of minimum variance among orderly estimates of location.*

(Again, the analogs with "to the left . . . the lefthandmost" or "in both directions . . . statistic," follow by symmetry.) For if y_j were of minimum variance, and y_n the righthandmost, then by

$$(B_1)\ \text{cov}\ (y_n - y_j, y_j) = \text{cov}\ (q_n - q_j, q_j) < 0,$$

and by (C) y_j is not of minimum variance. It is reasonable to anticipate that, actually, all coefficients must be positive (particularly for distributions with monotone scores), but the elementary methods used here do not seem to show this easily.

REFERENCES

[1] J. Berkson, "Question 21," *Amer. Statistician*, Vol. 3 (Oct., 1949), p. 12.

[2] R. A. Fisher, "Theory of statistical estimation," *Proc. Cambridge Philos. Soc.*, Vol. 22 (1923–25), pp. 700–725.

[3] C. Hastings, F. Mosteller, J. W. Tukey and C. P. Winsor, "Low moments for small samples: a comparative study of order statistics," *Ann. Math. Stat.*, Vol. 18 (1947), pp. 413–426.

[4] J. W. Tukey, "Non-parametric estimation, II. Statistical equivalent blocks and tolerance regions . . . the continuous case," *Ann. Math. Stat.*, Vol. 18 (1947), pp. 529–539.

AN ELEMENTARY THEOREM CONCERNING STATIONARY ERGODIC PROCESSES

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1. Introduction. The purpose of this note is to state and prove a theorem concerning strictly stationary, ergodic processes and to give some of its applications. Although the theorem itself is a simple consequence of the ergodic theorem, its applications include a proof of the consistency of the maximum likelihood estimates for stationary distributions and an extension of the zero-one law for symmetric sets given by Hewitt and Savage [1].

Theorem 1. *Let $\cdots x_{-1}, x_0, x_1, \cdots$ be a strictly stationary process such that every set invariant under shifts has measure zero or one. Let $\{\phi_n\}$ be a sequence of real-valued functions, ϕ_n being a measurable function of $n + 1$ variables. Then if the sequence $\phi_n(x_0, \cdots, x_n)$ and the sequence $\phi_n(x_{-n}, \cdots, x_0)$ both converge in probability, their limits are almost surely constant and equal.*

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