Using 1.6 and the linear independence of the B's, 2.1 yields

$$(2.2) \quad (c_0 I, c_1 I, \cdots, c_m I)\begin{bmatrix} (d_{00} - e)I & d_{01} I & \cdots & d_{0m} I \\ d_{10} I & (d_{11} - e)I & & d_{1m} I \\ \vdots & \vdots & & \vdots \\ d_{m0} I & d_{m1} I & \cdots & (d_{mm} - e)I \end{bmatrix} = 0.$$

Therefore

$$(2.3) \qquad (c_0, c_1, \cdots, c_m)(D - eI) = 0.$$

If C has m^* distinct non-zero characteristic roots, $e_1, e_2, \cdots, e_{m^*}$, then we may write

$$C = e_1E(e_1) + e_2E(e_2) + \cdots + e_{m^*}E(e_{m^*}).$$

Now using Theorem 2 we have

THEOREM 3. *The C matrix of* a P.B.I.B. (m) *may be expressed as a linear function of the* $m + 1$ *commutative and linearly independent matrices* $B_0, B_1, \cdots, B_m$.

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ON A FACTORIZATION THEOREM IN THE THEORY OF ANALYTIC CHARACTERISTIC FUNCTIONS[1]

Dedicated to Paul Lévy on the occasion of his seventieth birthday

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1. Introduction. Let $F(x)$ be a distribution function, that is, a non-decreasing right-continuous function such that $F(-\infty) = 0$ and $F(+\infty) = 1$. The characteristic function

$$(1.1) \qquad \phi(t) = \int_{-\infty}^{\infty} e^{itx}\, dF(x)$$

of the distribution function $F(x)$ is defined for all real t. A characteristic function is said to be an *analytic characteristic function* if it coincides with a regular analytic function $\phi(z)$ in some neighborhood of the origin in the complex z-plane.

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