ON THE LIMITING DISTRIBUTION OF THE NUMBER OF COINCIDENCES CONCERNING TELEPHONE TRAFFIC

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1. Introduction. Let us consider a telephone exchange. Suppose that the subscribers make calls at the instants $\tau_1, \tau_2, \cdots, \tau_n, \cdots$, where $\tau_n - \tau_{n-1}$ ($n = 1, 2, \cdots; \tau_0 = 0$) are identically distributed independent positive random variables with the distribution function $F(x)$. Put $\varphi(s) = \int_0^\infty e^{-sx}\, dF(x)$, $\alpha = \int_0^\infty x\, dF(x)$ and $\sigma^2 = \int_0^\infty (x - \alpha)^2\, dF(x)$.

Suppose that there is an infinite number of fully available channels and that each call gives rise to a connection (conversation) on one of the free channels. Denote by χ_n the duration of the holding time beginning in the instant τ_n ($n = 1, 2, \cdots$). It is assumed that χ_n ($n = 1, 2, \cdots$) are identically distributed mutually independent positive random variables, which are independent also of the random variables τ_n ($n = 1, 2, \cdots$). Suppose that $\mathbf{P}\{\chi_n \leqq x\} = 1 - e^{-\mu x}$, if $x \geqq 0$.

We say that the system is in state E_k ($k = 0, 1, 2, \cdots$) if k channels are busy. In what follows we shall deal with the determination of the distribution of the number of transitions $E_k \rightarrow E_{k+1}$ ($k = 0, 1, 2, \cdots$) occurring in the time interval $(0, t]$ and the corresponding asymptotic distribution as $t \rightarrow \infty$.

The above problem was solved earlier by the author [7] in the particular case when $\{\tau_n\}$ forms a Poisson process with density λ.

2. Notation. Denote by $\eta(t)$ the number of busy channels at the instant t and put

$$\mathbf{P}\{\eta(t) = k\} = P_k(t), \qquad (k = 0, 1, 2, \cdots).$$

Define the r-th binomial moment of $\eta(t)$ as follows:

$$B_r(t) = \sum_{k=r}^{\infty} \binom{k}{r} P_k(t), \qquad (r = 0, 1, 2, \cdots)$$

and put

$$\beta_r(s) = \int_0^\infty e^{-st} B_r(t)\, dt, \qquad (\Re(s) > 0).$$

Further denote by $\nu_t^{(k)}$ the number of transitions $E_k \rightarrow E_{k+1}$, ($k = 0, 1, 2, \cdots$), occurring in the time interval $(0, t]$. (We say that a transition $E_{-1} \rightarrow E_0$ takes place at $t = 0$.) Denote by $m_k(t)$ the expectation of the random variable $\nu_t^{(k)}$. ($m_{-1}(t) = 1$ if $t \geqq 0$ and $m_{-1}(t) = 0$ if $t < 0$.)

Finally denote by $m(t)$ the expectation of the number of calls taking place in

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