

e.g. on $2 \mid \Sigma \mid^{1/2} / \text{tr } \Sigma$. In the third place, it can be shown that the probability ratio is monotonic. This can be demonstrated either by starting from the Wishart distribution, or by using (2). However, in this example the latter way does not seem to be any simpler than the former. The moral seems to be that in some cases the utilization of the representation (1) or (2) leads to the results in a fast and elegant way, but in other cases the conventional approach may be more practical.

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ON DVORETZKY'S STOCHASTIC APPROXIMATION THEOREM

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1. Introduction. A very general theorem was proved by Dvoretzky [2] on the convergence of transformations with superimposed random errors. This work followed that of Robbins-Monro [5] and others (see [6] for bibliography) and contains the most comprehensive results on convergence (with probability one and in mean square) of the stochastic approximation procedures of Robbins-

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