BAYES SOLUTIONS OF THE STATISTICAL INVENTORY PROBLEM[1]

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1. Introduction. The inventory problem as discussed in the paper of Arrow, Harris, and Marschak [1] is a sequential decision problem. At the beginning of each time period a decision is made to stock a quantity of a specific item in anticipation of demand during that period. If the demand exceeds the available supply, the next period is begun with an initial stock which is either zero or negative. If the demand, on the other hand, is less than the available supply, the subsequent period begins with a stock level which is equal to the excess of supply over demand. In both cases the stock level may be augmented by additional purchasing.

A number of costs are assumed to be operative in this situation, among them, a cost of purchasing stock, a cost of holding or maintaining the stock in inventory, and a cost which arises whenever current inventory is insufficient to meet demand. The main problem is the determination of a sequence of purchasing or ordering decisions which minimizes some criterion built up from these costs. The criterion adopted in this paper is to discount costs incurred n periods in the future by an amount α^n, and to select that sequence of stockage decisions which minimizes the sum of all discounted costs. An elaborate discussion of the costs and the general structure of inventory models may be found in [2]. In the paper of Arrow, Harris, and Marschak and in a number of other papers in this area, the assumption has been made that the quantity demanded during any time period is a random variable whose distribution is known and unchanging from period to period. However, in the second of two papers by Dvoretsky, Kiefer and Wolfowitz [3] a more general situation in which the demand distribution is not known precisely is examined from the point of view of statistical decision theory. In the present paper our concern will be with this latter problem. The treatment, in distinction to that given in the paper by Kiefer, et al will be very specific, in the sense that we shall restrict our attention to very simple types of cost functions in order to obtain some detailed results about the optimal stockage policies.

The costs will be as follows:

(a) The ordering cost $c(z)$, as a function of the amount ordered, will be assumed to be linear, i.e., $c(z) = cz$.

(b) If the inventory at the close of the period is positive a holding cost $h(x)$ will be incurred, which in this paper is assumed to be a linear function of the quantity of inventory on hand at the end of the period, i.e., $h(x) = hx$.

(c) If the quantity of stock at the end of any period is negative a linear pen-

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