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A GENERALIZATION OF THE GLIVENKO-CANTELLI THEOREM

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A theorem referred to as the Glivenko theorem or the Glivenko-Cantelli theorem states that if X_1, X_2, $\cdots$, X_n, $\cdots$ is a sequence of independent, identically distributed random variables with any common distribution function $F(x)$, then the sequence $\{F_n(x)\}$ of empirical distribution functions converges uniformly to $F(x)$ with probability one. (See Loève [3] and Gnedenko [2].) The assumption of independence is not necessary for this theorem, and it is readily observed that the same conclusion holds if the sequence of random variables is a strictly stationary, ergodic (or metrically transitive) sequence. The purpose of this note is to prove a generalization of this theorem in the case where the sequence of random variables is strictly stationary, not necessarily ergodic, and with the same assumption that the common distribution function is arbitrary.

It is assumed that the reader is familiar with strictly stationary stochastic processes (with discrete time) and is acquainted with the notion of measure-preserving set transformation determined by the process and the notion of random variable transformation determined by this set transformation. Information on these concepts is available in Doob [1] and Loève [3]. The principal result to be used in the proof of the theorem is the ergodic theorem for random variables (see Loève [3], p. 434), which can be stated as follows:

> *Let S be a measure-preserving set transformation over the probability space $(\Omega, \mathcal{C}, P)$, let T be the random variable transformation determined by S, and let $\mathfrak{I}$ be the invariant sub-sigma-field of $\mathcal{C}$ determined by S. If X is any random variable for which $E \mid X \mid < \infty$, then*

$$P\{n^{-1}(X + TX + \cdots + T^{n-1}X) \to E(X \mid \mathfrak{I})\} = 1.$$

By means of the ergodic theorem in this form the following theorem is obtained.