NOTES

THE EXPRESSION OF k-STATISTIC k_{11} IN TERMS OF POWER SUMS AND SAMPLE MOMENTS

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The values of k_9 and k_{10} in terms of products of power sums s_r's ($s_r = \sum a^r$) have been published by the author ([7]). In this paper k_{11} is expressed in terms of s_r-products and sample moments, and is computed with the help of the tables of generalized k-statistics constructed by Abdel-Aty ([1]). From these tables

$$k_{11} = \frac{3628800}{n^{(11)}}[1^{11}] - \frac{19958400n}{n^{(10)}}[21^9] + \cdots \frac{[11]}{n},$$

where $n^{(11)} = n(n-1)(n-2)\cdots(n-10)$ and $[1^{11}]$, $[21^9]$ etc., are the augmented symmetric functions, which can be expressed in terms of s_r-products from the tables of symmetric functions given by David and Kendall [2]. Collecting all terms, k_{11} is expressed in terms of s_r-products.

As a check, the sum of the coefficients of all s_r's is $1/n$.

k_{11} is obtained in terms of sample moments m_r' by putting $s_1 = 0$ and $s_r = nm_r$ ($r > 1$). Thus $s_{11} = nm_{11}$, $s_3s_2^4 = nm_3(nm_2)^4 = n^5m_2^4m_3$, etc.

The k-statistics have recently been applied in various fields by several writers such as Tukey [6], Hooke [4], Robson [5]. They are of interest to workers in the theory of sampling distributions and moment statistics. They are related also to certain aspects of the theory of numbers and combinatory analysis, as indicated by Dressel [3].

$$\begin{aligned}
k_{11} = \frac{1}{n^{(11)}}&[3628800s_1^{11} - 19958400ns_2s_1^9\\
&+ 39916800(n^2-n)s_2^2s_1^7 - 34927200(n^3-3n^2+2n)s_2^3s_1^5\\
&+ 12474000(n^4-6n^3+11n^2-6n)s_2^4s_1^3\\
&- 1247400(n^5-10n^4+35n^3-50n^2+24n)s_2^5s_1 + 6652800(n^2+8n)s_3s_1^8\\
&- 23284800(n^3+5n^2-6n)s_3s_2s_1^6\\
&\qquad + 24948000(n^4+n^3-10n^2+8n)s_3s_2^2s_1^4\\
&- 8316000(n^5-4n^4-n^3+16n^2-12n)s_3s_2^3s_1^2\\
&+ 415800(n^6-10n^5+35n^4-50n^3+24n^2)s_3s_2^4\\
&+ 3326400(n^4+8n^3+25n^2-34n)s_3^2s_1^5
\end{aligned}$$

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