ASYMPTOTIC EXPANSIONS IN GLOBAL CENTRAL LIMIT THEOREMS

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1. Introduction. Let $\xi_1, \xi_2, \cdots$ be independent random variables having the same d.f. (distribution function) $F(x)$. We suppose that

$$\int_{-\infty}^{\infty} x\, dF(x) = 0, \qquad \int_{-\infty}^{\infty} x^2\, dF(x) = 1 \tag{1.1}$$

so that $F(x)$ has mean 0 and standard deviation 1. Let $F_n(x)$ denote the d.f. of the normalized sum

$$(\xi_1 + \xi_2 + \cdots + \xi_n)/n^{\frac{1}{2}}. \tag{1.2}$$

A special case of the central limit theorem then asserts that, for each individual x in the interval $-\infty < x < \infty$,

$$\lim_{n\to\infty} F_n(x) = \Phi(x) \tag{1.21}$$

where $\Phi(x)$ is the Gaussian d.f. defined by

$$\Phi(x) = (2\pi)^{-\frac{1}{2}} \int_{-\infty}^{x} e^{-u^2/2}\, du. \tag{1.22}$$

It is our purpose to study the behavior as $n \to \infty$ of the constants C_n defined by

$$C_n = \int_{-\infty}^{\infty} |F_n(x) - \Phi(x)|^2\, dx. \tag{1.3}$$

For each $p > 0$, let constants $C_n(p)$ be defined by

$$C_n(p) = \int_{-\infty}^{\infty} |F_n(x) - \Phi(x)|^p\, dx \tag{1.31}$$

when these integrals exist, that is, are finite. It is known from [1] and [2] that the hypotheses (1.1) imply that if $p > \frac{1}{2}$ then the constants $C_n(p)$ exist and $\lim_{n\to\infty} C_n(p) = 0$. Beyond this, not very much is known about the constants $C_n(p)$. The moments α_k and the absolute moments β_k of $F(x)$ are defined by

$$\alpha_k = \int_{-\infty}^{\infty} x^k\, dF(x), \qquad \beta_k = \int_{-\infty}^{\infty} |x|^k\, dF(x) \tag{1.4}$$

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