ABSTRACTS OF PAPERS

(Abstracts of papers not presented at any meeting of the Institute)

6. **On a χ^2-Test with Cells Determined by Order Statistics.** HERMANN WITTING, University of Freiburg. (By title)

Let $X_1, \cdots, X_n$ be a sample of a continuous one-dimensional probability distribution $Q(A)$; let $X_{n,1}, \cdots, X_{n,k-1}$ be order statistics for given ranks $r_{n,j}$ with $p_{n,j} = (r_{n,j} - r_{n,j-1})/(n+1) = p_j + o(1/\sqrt{n})$. Let $S_{n,j} = \{x: X_{n,j-1} < x \leqq X_{n,j}\}$. For testing the hypothesis that $Q(A)$ belongs to an s-parametric class of probability distributions $P(A, \theta)$ the test statistic $T_n = \sum_{j=1}^{k} n(P(S_{n,j}, \hat{\theta}_n) - p_{n,j})^2/p_{n,j}$ is used, where $\hat{\theta}_n$ is the minimum-χ^2-estimate. Then if $Q(A) = P(A, \theta_0)$ or $Q(A) = P(A, \theta_0) - q(A)/\sqrt{n}$, respectively, under certain regularity conditions T_n is asymptotically distributed as χ^2 with $(k - s - 1)$ degrees of freedom (and noncentrality parameter $\sum_{j=1}^{k} q_j^2/p_j$, $q_j = p\text{-lim } q(S_{n,j})$. Using $(k-1)$ continuous functions $\varphi_1(x), \cdots, \varphi_{k-1}(x)$, defining $\varphi_j(X_{n,j})$ successively by ordering the values $\varphi_j(X_i)$ and defining $S_{n,j} = \{x: \varphi_l(x) > \varphi_l(X_{n,l}), l = 1, \cdots, j-1; \varphi_j(x) \leqq \varphi_j(X_{n,j})\}$, the same limiting behaviour of T_n holds for probability distributions in a metric space. The proof is based on the fact that the $Q(S_{n,j})$ are jointly B-distributed (cf. J. W. Tukey, *Ann. Math. Stat.* 18(1947)529). Therefore $\sqrt{n}(Q(S_{n,j}) - p_{n,j})$ are asymptotically $N(0, C)$ where C is of rank $(k-1)$ and coincides with the covariance matrix of the multinomial distribution, underlying the corresponding classical χ^2-test with the cells $S_j = p\text{-lim } S_{n,j}$. While having the same power, this modified χ^2-procedure has certain advantages over the classical χ^2-test.

7. **A Generalized Pitman Efficiency for Nonparametric Tests.** HERMANN WITTING, University of Freiburg. (By title)

Asymptotic expressions up to terms of order n^{-2} are given for the efficiency of the Wilcoxon two-sample test relative to the $\bar{x}$- and t-tests for nearby alternatives. The first term is the well-known Pitman efficiency; the remaining terms are corrections for finite sample sizes. Efficiency values are given for finite sample sizes in the case of normal and rectangular distributions and comparisons with the exact values are made. In general the Wilcoxon test is shown to be nearly as good locally for moderate sample sizes as it is known to be asymptotically. A similar analysis is performed for the single-sample sign test.

(Abstracts of papers to be presented at the Washington, D. C., Annual Meeting of the Institute, December 27–30, 1959. Additional abstracts will appear in the March, 1960 issue.)

1. **Some Nonparametric Problems: I.** V. P. BHAPKAR, University of North Carolina and University of Poona. (By title)

Mood and Brown have considered a nonparametric test for the equality of row effects in the two-way classification with one observation per cell or the same number of observations per cell. In this paper, first their test has been extended to cover incomplete block situations. For the BIBD in the usual terminology, if m_i denotes the number of observations, for the ith 'treatment', that exceed the respective 'block'-medians, then to test the equality of 'treatment'-effects we have $(k^2(k-1))/(a(k-a)\lambda v)\ \sum_{i=1}^{v}(m_i - (ra/k))^2$ asymptotically distributed as a χ^2 with $v - 1$ d.f. for large r, where a is $k/2$ if k is even and $(k-1)/2$ otherwise. The χ^2 statistic appropriate for PBIBD is also given.

Next, Hoeffding's theorem on U-statistics extended by Lehmann to the case of two samples, has been extended to the case of c samples. This is then applied to derive a new