A NOTE ON MULTIPLE INDEPENDENCE UNDER MULTIVARIATE NORMAL LINEAR MODELS

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1. Introduction. S. N. Roy and Bargmann [3] used S. N. Roy's union-intersection method as the basis for providing tests and confidence intervals in the following cases:

i) $\mathbf{y}' = (y_1, \cdots, y_p) \sim N(\boldsymbol{\mu}', \Sigma)$, $H_o: \sigma_{ij} = 0, i \neq j$.
ii) $\mathbf{y}' \sim N(\boldsymbol{\mu}', \Sigma)$, but $\mathbf{y}'$ is partitioned into k sets or blocks or sizes $p_1, \cdots, p_k$. $H_o: \Sigma_{ij} = 0$, $i \neq j$, where Σ_{ij} is the covariance matrix between blocks i and j.

J. Roy [1] considered the following additional cases:

iii) $Y: n \times p$, $(y_{1j}, \cdots, y_{pj}) \sim N(—, \Sigma)$, $j = 1, \cdots, n$, $EY = A\theta$. $A: n \times m$ has rank $r \leqq n - p$ and is known, θ is unknown. Let $\Phi = B\theta$ be estimable, $B: t \times m$. $H_o: \Phi = 0$.
iv) $(y_1, \cdots, y_p) \sim N(\boldsymbol{\mu}', \Sigma)$. $H_o: \Sigma = \Sigma_0$ (specified).
v) $(y_1, \cdots, y_p) \sim N(\boldsymbol{\mu}', \Sigma_1)$, $(x_1, \cdots, x_p) \sim N(\boldsymbol{\nu}', \Sigma_2)$, $H_o: \Sigma_1 = \Sigma_2$.

In this note we shall consider the following modification of (iii):

vi) $Y: n \times p$, $(y_{1j}, \cdots, y_{pj}) \sim N(—, \Sigma)$, $j = 1, \cdots, n$, $EY = A\theta$ (as in (iii)). $H_o: \sigma_{ij} = 0$, $i \neq j$.

2. Step-down procedure to test H_o in (vi). In the notation of [1], denote the ith columns of the matrices Y and θ by $\mathbf{y}_i$ and $\boldsymbol{\theta}_i$ respectively and write

$$Y_i = [\mathbf{y}_1, \cdots, \mathbf{y}_i], \qquad \theta_i = [\boldsymbol{\theta}_1, \cdots, \boldsymbol{\theta}_i]$$

and $\Sigma_i = (\sigma_{jk})$, $j, k = 1, \cdots, i$.

If Y_i is fixed, the n elements of $\mathbf{y}_{i+1}$ are distributed independently and normally with the same variance σ_{i+1}^2 and expectations given by

$$E(\mathbf{y}_{i+1} \mid Y_i) = A\boldsymbol{\eta}_{i+1} + Y_i\boldsymbol{\beta}_i, \tag{1}$$

where $\boldsymbol{\beta}_i': 1 \times i$ is the row vector,

$$\boldsymbol{\beta}_i' = (\sigma_{1,i+1}; \cdots; \sigma_{i,i+1})\Sigma_i^{-1}, \tag{2}$$

and $\boldsymbol{\eta}_{i+1}: m \times 1$ is the column vector given by

$$\boldsymbol{\eta}_{i+1} = \boldsymbol{\theta}_{i+1} - \theta_i\boldsymbol{\beta}_i, \qquad i = 1, \cdots, p - 1. \tag{3}$$

We note that H_o is true if and only if the hypothesis $H_i: \boldsymbol{\beta}_i = 0$ holds for all $i = 1, \cdots, p - 1$. Now the elements of the vectors $\boldsymbol{\beta}_i$ and $\boldsymbol{\eta}_{i+1}$ may be regarded

Received December 24, 1958; revised May 11, 1959.