THIRD ORDER ROTATABLE DESIGNS FOR EXPLORING RESPONSE SURFACES[1, 2]

By D. A. Gardiner

Oak Ridge National Laboratory

AND

A. H. E. Grandage and R. J. Hader

North Carolina State College

1. Introduction. This paper considers a problem arising in the design of experiments for empirically investigating the relationship between a dependent and several independent variables, all variables being continuous. It is assumed that the form of the functional relationship is unknown but that within the range of interest, the function may be represented by a Taylor series expansion of moderately low order. Specifically, the problem considered herein is that choice of combinations of levels of the independent variables which, a) will enable an experimenter to approximate a functional relationship by fitting a Taylor series expansion through terms of order 3, by the method of least squares, and b) will have the property of rotatability. Such a choice of combinations of levels of the independent variables will be called a third order rotatable design.

For the sake of brevity, the abbreviation dth ORD will be used to denote dth order rotatable design.

2. Rotatability. The property of rotatability as a desirable quality of an experimental design was first advanced by Box and Hunter in [1]. This property is that the variances of estimates of the response made from the least squares estimates of the Taylor series are constant on circles, spheres or hyper-spheres about the center of the design. Thus, a rotatable design, that is, a design which achieves this property, could be rotated through any angle around its center and the variances of responses estimated from it would be unchanged.

Box and Hunter proved that a necessary and sufficient condition for a design of order d ($d = 1, 2, 3, \cdots$) to be rotatable is that the moments of the independent variables be the same, through order $2d$, as those of a spherical distribution, or that these moments be invariant under a rotation of the design around its center.

Let k be the number of independent variables, or factors, and let $x_{1u}, x_{2u}, \cdots, x_{ku}$ be the levels of these variables for the uth experimental point in the factor space, ($u = 1, 2, \cdots, N$). Then a pth order moment is defined as

$$N^{-1} \sum_{u=1}^{N} x_{1u}^{q} x_{2u}^{r} \cdots x_{ku}^{t},$$

Received June 16, 1958; revised March 16, 1959.

[1] This work was sponsored in part by the U. S. Army under Contract No. DA-36-034-ORD-1517(RD) (Experimental Designs for Industrial Research).

[2] This paper is based on a part of the Ph.D. Thesis (N. C. State College, 1956) by Gardiner [2].