BAYES ACCEPTANCE SAMPLING PROCEDURES FOR LARGE LOTS[1]

By D. Guthrie, Jr. and M. V. Johns, Jr.

Stanford University

1. Introduction and statement of the main results. A lot consisting of N items may be characterized by N non-negative random variables X_i, $i = 1, 2, \cdots, N$, where the value of X_i indicates the quality of the ith item. In a typical case X_i might take on the values zero and one according to whether the ith item is non-defective or defective. Alternatively, X_i might be defined to be the number of defects in the ith item so that the possible values of X_i would be 0, 1, 2, $\cdots$. In still another formulation X_i might be a continuous random variable related to the deviation from standard of some characteristic of the item. We shall assume that the random variables X_i, $i = 1, 2, \cdots, N$, are independent and identically distributed with common distribution function $F(x \mid \lambda)$ depending on a single parameter λ.

The fixed size sampling inspection scheme to be considered consists of the random selection of n items from the lot and the observation of the values of the corresponding X_i's. Thus, the sample may be described by the random variables $X_1, X_2, \cdots, X_n$. The two possible actions to be taken on the basis of the sample are acceptance or rejection of the uninspected remainder of the lot. The consequences of these alternative actions are appraised by the following cost model where we let $S_k = \sum_{i=1}^{k} X_i$ for any $k = 1, 2, \cdots, N$:

(1.1)

Action	Cost
Acceptance	$a_1(S_N - S_n) + a_2(N - n) + s_1 S_n + s_2 n$
Rejection	$r_1(S_N - S_n) + r_2(N - n) + s_1 S_n + s_2 n.$

Thus, for $i = n + 1, n + 2, \cdots, N$, the contributions to the total cost due to the acceptance or rejection of the ith item without inspection are given by $a_1 X_i + a_2$ and $r_1 X_i + r_2$ respectively. For $i = 1, 2, \cdots, n$ the cost of inspection (and possibly replacement) of the ith item is given by $s_1 X_i + s_2$. If, for example, an item is classified as defective or non-defective by X_i, then S_N and S_n are the number of defective items in the lot and in the sample respectively. Suppose that the cost of accepting an item is a_1 if the item is defective and zero if the item is non-defective, and that the cost of rejecting the uninspected remainder of the lot is proportional to the number of items remaining in the lot. Then $a_2 = r_1 = 0$. If, in addition, all items found to be defective in the sample are replaced with good items, each at a cost of s_1 units, and s_2 represents the cost of the time and labor required to inspect each item in the sample, then (1.1)

Received June 16, 1958.

[1] Prepared for Army, Navy, and Air Force under Contract N6onr25126 with the Office of Naval Research. Reproduction in whole or part permitted for any U. S. Government purpose.