ABSTRACTS OF PAPERS

(Abstract of a paper presented at the Cambridge, Massachusetts Meeting of the Institute, August 25–28, 1958.)

31. **Markov Renewal Processes.** Ronald Pyke, Columbia University. (Invited Paper presented under the title, "On Multi-event Renewal Processes.")

Let $\mathcal{Q} = \| Q_{ij} \|$, $1 \leqq i, j \leqq m$, $m < \infty$ be a matrix of transition distributions, i.e. each Q_{ij} is a mass function satisfying $Q_{ij}(t) = 0$ for $t \leqq 0$ and $\sum_{j=1}^{m} Q_{ij}(+\infty) = 1$. For discrete probabilities $a_1, \cdots, a_m$, let $\{(J_n, X_n); n \geqq 0\}$ be a stochastic process satisfying

$$P[J_0 = k] = a_k, \qquad X_0 = 0,$$

and $P[J_n = k, X_n \leqq x \mid J_0, J_1, X_1, \cdots, J_{n-1}, X_{n-1}] = Q_{J_{n-1},k}(x)$ a.s. For $t \geqq 0$, $1 \leqq j \leqq m$, define $N_j(t)$ as the number of times $J_n = j$ and $S_n \leqq t$ for $n > 0$, where

$$S_n = X_1 + \cdots + X_n .$$

The vector process $\{N_1(t), \cdots, N_m(t); t \geqq 0\}$ is called a Markov Renewal Process (M.R.P.). Alternatively, it is possible to define an M.R.P. as an equivalent 1-dimensional process. Set $N(t) = N_1(t) + \cdots + N_m(t)$, and define $Z_t = J_{N(t)}$. The process

$$\{Z_t : t \geqq 0\}$$

is called a Semi-Markov Process (S.-M.P.). An M.R.P. is an S.-M.P. (with a finite state space) if and only if $Q_{ii} \equiv 0$ for all i. Let $P_{ij}(t) = P[Z_t = j \mid Z_0 = i]$ and

$$G_{ij}(t) = P[N_j(t) > 0 \mid Z_0 = i],$$

the latter being the first passage-time distribution from state i to state j. Relationships between the Q_{ij}, P_{ij} and G_{ij} are derived. These can be solved to obtain expressions for the P_{ij} and G_{ij} in terms of the Q_{ij}. For example, one may show $\| P_{ij} \| = (I - \mathcal{Q})^{(-1)}(I - \mathcal{H})$ where the elements of $\mathcal{H}$ are given by $H_{ij} = \delta_{ij} \sum_{k=1}^{m} Q_{ik}$. M.R.P.'s are generalizations both of discrete and continuous parameter Markov Chains. They have many applications, the one which motivated the author's definition and study of these processes being to the theory of multiple channel electronic counters.

(Abstracts of papers presented at the Washington, D. C., Annual Meeting of the Institute, December 27–30, 1959.)

24. **Main-Effect Designs for Asymmetrical Factorial Experiments.** Sidney Addelman, Iowa State University.

A method of constructing orthogonal designs which allow the estimation of main effects for a general class of asymmetrical factorial experiments is presented. By the use of the suggested method of construction, it is possible to obtain a design in which all main effects are preserved, for the $s_1^{t_1} \times s_2^{t_2} \times \cdots \times s_k^{t_k}$ experiment in s_1^n observations, where s_1 is a prime or a power of a prime, $s_1 > s_2 > \cdots > s_k$, and $\sum_{i=1}^{k} t_i = (s_1^n - 1)/(s_1 - 1)$. As an interesting consequence of the above method of construction, one is able to obtain main-effect designs for symmetrical factorial experiments in which the number of levels of each factor is not a prime or a power of a prime.