# ON THE MEDIAN OF THE DISTRIBUTION OF EXCEEDANCES

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The distribution of exceedances may be defined by the following formula (see, e.g. [2]), where $x$ corresponds to the number of exceedances

$$(1)\quad w(n_1, m, n_2, x) = \frac{\binom{n_1 + n_2 - m - x}{n_1 - m}\binom{x + m - 1}{m - 1}}{\binom{n_1 + n_2}{n_1}},$$

$$x = 0, 1, \cdots, n_2; n_1, n_2, m \leqq n$$

given natural numbers.

There are known—among others—the following two fundamentally equivalent models or representations of this distribution [2], [3], [4]:

*A. Exceedances.* We have two random samples of sizes $n_1$ and $n_2$, respectively, from the same continuous distribution. The number of exceedances is defined as the number of elements of the second sample which surpass at least $n_1 - m + 1$ elements of the first, for a fixed natural number $m \leqq n_1$. The distribution of the number of exceedances is given by formula (1).

*B. Pascal model without replacement.* An urn contains $n_1$ black and $n_2$ red balls. We draw balls from the urn until we have drawn $m$ black balls. The distribution of the number of the red balls drawn is given by (1).

In [1], Gumbel proved that, for $n_1 = n_2$, the median of the number of exceedances is $m - 1$, more precisely that

$$(2)\qquad W(n,m,n,m - 1) = \tfrac{1}{2},$$

where $W$ is the cumulated form of $w$,

$$W(n_1, m, n_2, x) = \sum_{y=0}^{x} w(n_1, m, n_2, y)$$

In this paper a simple proof of this result is given.

We shall, in fact, prove the following more general result:

$$(3)\qquad W(n_1, m_1, n_2, m_2 - 1) + W(n_2, m_2, n_1, m_1 - 1) = 1.$$

In terms of model $A$, the $m_1$th element of the first sample exceeds the $m_2$th element of the second sample if and only if the number of exceedances ($y$) takes one of the values $0, 1, \cdots, m_2 - 1$, these possibilities being mutually exclusive.

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