

# NOTES

## NOTE ON THE DISTRIBUTION OF LOCALLY MAXIMAL ELEMENTS IN A RANDOM SAMPLE<sup>1</sup>

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Glasgow's formula for the second factorial moment of this distribution [1] is considerably more complicated than it need be. We have elsewhere [2] and [3] obtained a formula requiring just one summation, over the fixed range  $0 \leq s \leq h - 1$ , thus eliminating the summation over the ever-increasing range  $0 \leq s \leq m$ .

Following Glasgow's notation, let  $\beta$  be the number of locally  $k$ -maximal elements in a permutation of the first  $n$  integers. Our formula, for the variance of  $\beta$ , is

$$\text{var}(\beta) = (n + 1)C_k, \quad n \geq 2k,$$

where

$$C_k = \frac{-2k(5k + 3)}{(2k + 1)(k + 1)^2} + \frac{8}{k + 1} \sum_{s=0}^{k-1} \frac{1}{k + s + 2}.$$

Using the expected value of  $\beta$  given in [1], we find that

$$\begin{aligned} E(\beta^{(2)}, n) &= \text{var}(\beta) + E(\beta)(E(\beta) - 1) \\ &= (n + 1)C_k + (2n - k + 1)(2n - 2k)/(k + 1)^2. \end{aligned}$$

Both referees have pointed out that Glasgow's formula can be reduced to ours. In fact, the summation in his equation (3.8) can be performed, yielding

$$\frac{2(m + 1)(7k^2 + 10k + 3 + 4km + 2m)}{(k + 1)^2(2k + 1)(2k + m + 2)}.$$

### REFERENCES

- [1] M. O. GLASGOW, "Note on the factorial moments of the distribution of locally maximal elements in a random sample," *Ann. Math. Stat.*, Vol. 30 (1959), pp. 586-90.
- [2] M. FREIMER, B. GOLD, AND A. L. TRITTER, "On a mathematical model for a Morse code translator," Lincoln Laboratory Group Report 34-61, November 1, 1957. (Not generally available.)
- [3] M. FREIMER, B. GOLD, AND A. L. TRITTER, "The Morse distribution," *IRE Transactions on Information Theory*, IT-5 (1959), pp. 25-31.

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