NOTES

ON THE UNBIASEDNESS OF YATES' METHOD OF ESTIMATION USING INTERBLOCK INFORMATION[1]

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In a balanced incomplete block model with blocks and errors random normal variables, Yates has shown that there are two independent unbiased estimates for any treatment contrast. These are referred to as intrablock and interblock estimators. Yates has also given a method for combining these two estimators which depends on the variances (unknown) and has shown how to estimate the variances from an analysis of variance [1]. Since this combined estimator is used quite extensively, it seems desirable to study its properties. Graybill and Weeks [2] have shown that Yates' combined estimator is based on a set of minimal sufficient statistics and have presented an estimator which is unbiased.

The purpose of this note is to show that Yates' estimator, which is based on intrablock and interblock information, is unbiased.

The model and distributional assumptions in this paper are exactly those given in [2], and the same notations are used and will not be repeated here.

In [2] it is shown that Yates' estimator (denoted by $\bar{\tau}_i$) of τ_i is

$$(1) \qquad \begin{aligned} \bar{\tau}_i &= x_i + \gamma(u_i - x_i) && \text{if} \quad \hat{\sigma}_\beta^2 > 0 \\ &= x_i + \lambda t/rk(u_i - x_i) && \text{if} \quad \hat{\sigma}_\beta^2 \leqq 0 \end{aligned}$$

where

$$(2) \quad \gamma = \frac{\dfrac{\lambda^2 t(r-\lambda)}{rk(r-1)}(U-X)'(U-X) + \dfrac{\lambda k}{(r-1)} S^{*2} + \dfrac{\lambda(k-t)}{f(r-1)} S^2}{\dfrac{\lambda^2 t(r-\lambda)}{rk(r-1)}(U-X)'(U-X) + \dfrac{\lambda k}{(r-1)} S^{*2} + \left[\dfrac{\lambda(k-t)}{f(r-1)} + \dfrac{(r-\lambda)}{f}\right] S^2}$$

and where

$$(3) \quad \hat{\sigma}_\beta^2 = 1/t(r-1)[\lambda t(r-\lambda)/rk^2(U-X)'(U-X) + S^{*2} - (b-1)/fS^2]$$

We now define $\phi(\hat{\sigma}_\beta^2)$ such that

$$\begin{aligned} \phi(\hat{\sigma}_\beta^2) &= 0 && \text{if} \quad \hat{\sigma}_\beta^2 > 0 \\ &= 1 && \text{if} \quad \hat{\sigma}_\beta^2 \leqq 0 \end{aligned}$$

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