ABSTRACTS OF PAPERS

(Abstracts of papers presented at the Stanford Annual Meeting of the Institute, August 23–26, 1960.)

39. On the Number of Distinct Values in a Large Sample from an Infinite, Discrete Distribution. R. R. Bahadur, Indian Statistical Institute (By title).

Let $A_1, A_2, \cdots$ be an infinite sequence of events defined on the sample space of some experiment such that $P(A_j) > 0$ for each j, $P(A_j A_k) = 0$ for $j \neq k$, and $\sum_j P(A_j) = 1$. Consider a sequence of independent repetitions of the experiment, and let T_n be the number of distinct events A_j observed in the first n trials. The paper studies the rate at which $T_n \to \infty$ as $n \to \infty$. Let $\mu_n = E(T_n)$, where E denotes expected value. *Theorem* 1: $T_n/\mu_n \to 1$ in probability as $n \to \infty$. Suppose (with no loss in generality) that $P(A_j) \geqq P(A_{j+1})$ for all j. Let $f(x) = \max \{j : P(A_j) \geqq x\}$ for $x \leqq P(A_1)$ and $f(x) = 0$ (say) otherwise. *Theorem* 2: $\mu_n = n\int_0^\infty e^{-nx} f(x)\, dx + o(1)$ as $n \to \infty$. It follows, e.g., that if $P(A_j) = cj^{-\alpha}$, where $1 < \alpha < \infty$, then $\mu_n = \Gamma(1 - \beta)(cn)^\beta - \theta_n + o(1)$, where $0 \leqq \theta_n \leqq 1$ and $\beta = 1/\alpha$; and that if $P(A_j) = e^{-\lambda}\lambda^j/j!$, where $0 < \lambda < \infty$, then $\mu_n \sim \log n/\log \log n$. There is, however, no attainable maximum or minimum rate of increase of μ_n. *Theorem* 3: Given P, there exist probability distributions P^* and P^{**} such that, with $\mu_n^* = E(T_n \mid P^*)$ and $\mu_n^{**} = E(T_n \mid P_n^{**})$, $\mu_n^* = o(\mu_n)$ and $\mu_n = o(\mu_n^{**})$ as $n \to \infty$.

40. Expansions for Convolutions. Reed Dawson, American Systems Inc.

Asymptotic expansions for the ordinate and tail area in the distribution of the standardized sum of a large number of independent and identically distributed random variables are developed from the Edgeworth series of Cramér. The formula for the ordinate extends results of Daniels (*Ann. Math. Stat.*, Vol. 25 (1954), pp. 631–650) and Good (*Ann. Math. Stat.*, Vol. 28 (1957), pp. 861–881). The expansion of the tail area is believed to be new.

41. On Sufficient Conditions for Consistent Parameter-Estimates in a Stochastic Difference Equation with Regression on Several Lagged and Non-Stochastic Variables. Friedhelm Eicker, University of North Carolina. (By title).

The least squares estimates a_i of α_i and b_i of β_i in the stochastic difference equation $y_t = \alpha_1 y_{t-1} + \cdots + \alpha_p y_{t-p} + \beta_1 x_{1t} + \cdots + \beta_q x_{qt} + \epsilon_t$, $t = 1, 2, \cdots$, where $y_0, y_{-1}, \cdots, y_{-p+1}$ and all x_{it} are given constants, are shown to be consistent if conditions (A)–(C2) hold: (A) The disturbances ϵ_t are independent with 0 means, and 2nd and 4th moments bounded between two positive constants uniformly in t. (B) The roots of $\rho^p - \alpha_1\rho^{p-1} - \cdots - \alpha_p = 0$ are within the unit-circle (stationarity). With $X(N, q) = (x_{jt})$ and $\lambda_1(P) =$ largest eigenvalue of $P = X'X$ it is assumed further: (C1) If the set $\{N\}$ of those N for which $N^{-2}\lambda_1(P) \geqq g(N)$, with some $g(N) \to 0$ as $N \to \infty$, is infinite, let

$$(N + N[\lambda_1(P)]^{\frac{1}{2}} + \lambda_1(P))^{-\frac{1}{2}} \cdot \lambda_{\min}(K'K) \to \infty, \qquad N \text{ in } \{N\},$$

where $K(N, (p+1)q) = (X, LX, \cdots, L^pX)$, $L(N, N)$ containing 1's in its left subdiagonal and 0 elsewhere. (C2) If the complement $\{\bar{N}\}$ to $\{N\}$ is infinite, $\lambda_{\min}(P) \to \infty$ suffices for $a_i \to \alpha_i$ i.p. on $\{\bar{N}\}$. For $b_i \to \beta_i$ on $\{\bar{N}\}$ suffices to have in addition that the elements of $P^{-1}X'L^jX$ are uniformly bounded for all j. This theorem is proved using only elementary matrix theory. Simplified conditions are easily derived. They allow even exponentially in-