STATISTICAL PROGRAMMING[1]

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1. Introduction. A "statistical programming" problem is encountered when the information about one or more constants in a programming problem is statistical. We shall first give examples of programming problems and then point out how certain statistical analogues of them arise. The results given in this paper pertain to these analogues.

Our first example is a transportation problem. Given a unit amount of a homogeneous product (e.g., oil) at each of n origins and required that a unit amount be received at each of n destinations, and given the cost, say c_{ij}, of shipping a unit amount from the ith origin to the jth destination ($i, j = 1, \cdots, n; n \geqq 2$), find a most economical schedule of shipments of the product from origins to destinations. More specifically, find an $n \times n$ matrix (x_{ij}) of real numbers for which

$$\sum_{i,j=1}^{n} c_{ij}x_{ij} \tag{1}$$

assumes its *minimum* value, where

$$\begin{aligned} \sum_{i=1}^{n} x_{ij} &= 1, \qquad (j = 1, \cdots, n), \\ \sum_{j=1}^{n} x_{ij} &= 1, \qquad (i = 1, \cdots, n), \\ x_{ij} &\geqq 0. \end{aligned} \tag{2}$$

x_{ij} represents the amount shipped from the ith origin to the jth destination; and the matrix (x_{ij}) is called a "program." The expression in (1) is the total shipping cost. The condition (2) expresses the facts that at each origin the sum of all amounts shipped away must equal 1 and that at each destination the sum of all amounts received from the origins must equal 1. The problem stated above is a special case of the Hitchcock-Koopmans transportation problem, which is a well-known special case of a linear programming problem (see [1], [2, Part 1]).

The next example is the personnel assignment problem, which is closely related to the first example (see [4], [5, pp. 255–258], and [8]). Let us replace "origins" by "persons," "destinations" by "jobs," and regard c_{ij} as the productivity of the ith person if placed on the jth job. It is required that each person be

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