

A THIRD ORDER ROTATABLE DESIGN IN FOUR DIMENSIONS¹

BY NORMAN R. DRAPER

Mathematics Research Center, United States Army, Madison, Wisconsin

1. Introduction. Only one third order rotatable design in four dimensions is known, namely the 128 point design presented by Gardiner, Grandage and Hader [2]. This is formed by combining the 96 points

$$(\pm c, \pm c, \pm d, \pm d), (\pm c, \pm d, \pm c, \pm d), (\pm c, \pm d, \pm d, \pm c),$$

$$(\pm d, \pm c, \pm c, \pm d), (\pm d, \pm c, \pm d, \pm c), (\pm d, \pm d, \pm c, \pm c),$$

of the "truncated cube (2)" with the 16 points arising from two cross-polytopes

$$(\pm c_i, 0, 0, 0), (0, \pm c_i, 0, 0), (0, 0, \pm c_i, 0), (0, 0, 0, \pm c_i),$$

$i = 1, 2$, and the 16 points of a measure polytope

$$(\pm a, \pm a, \pm a, \pm a),$$

where a, c_i, c and d take appropriate values obtained after considerable computation. □

We now give a very simple design which requires 96 points (32 less) and is a combination of four second order rotatable arrangements, each containing 24 points. This permits the use of four convenient blocks of equal size or, alternatively, sequential performance in several ways. The notation and definitions of reference [1] are used in obtaining this design.

2. The construction of the design. Consider in four dimensions (a) the 24 points

$$(\pm p, \pm p, 0, 0), (\pm p, 0, \pm p, 0), (\pm p, 0, 0, \pm p),$$

$$(0, \pm p, \pm p, 0), (0, \pm p, 0, \pm p), (0, 0, \pm p, \pm p),$$

which we shall denote by $S(p, p, 0, 0)$; (b) the 8 points

$$(\pm c, 0, 0, 0), (0, \pm c, 0, 0), (0, 0, \pm c, 0), (0, 0, 0, \pm c),$$

which we shall denote by $S(c, 0, 0, 0)$; (c) the 16 points

$$(\pm a, \pm a, \pm a, \pm a),$$

which we shall denote by $S(a, a, a, a)$.

Applying the formulae for the various excess functions (as defined in [1]) for these sets, we find the results shown in Table I.

Received December 9, 1959.

¹ This research was supported at the University of North Carolina by the United States Air Force through the Air Force Office of Scientific Research of the Air Research and Development Command, under Contract No. AF 18(600)-83. Reproduction in whole or part is permitted for any purpose of the United States Government.