THE TRANSIENT BEHAVIOUR OF A COINCIDENCE VARIATE IN TELEPHONE TRAFFIC

By P. D. Finch[1]

University of Melbourne

1. Introduction. We consider the following problem. Calls arrive at a telephone exchange at the instants $t_1, t_2, \cdots, t_n$, where the inter-arrival intervals $(t_n - t_{n-1})$, $n \geqq 1$, $t_0 = 0$, are independently and identically distributed non-negative random variables with common distribution function $A(x)$ and finite expectation $\alpha = \int_0^\infty x\, dA(x)$. Introduce the Laplace-Stieltjes transform $a(s)$ defined by

$$a(s) = \int_0^\infty e^{-sx}\, dA(x). \tag{1}$$

There are m channels available and a connection is realised if the incoming call finds an idle channel. If all the channels are busy, then the incoming call is lost. Denote by β_n the holding time of the call at t_n if that call is not lost. We suppose that the β_n are non-negative independent random variables, independent also of the input process $\{t_n\}$, with common distribution function $B(x)$ given by

$$B(x) = 1 - e^{-\mu x}, \qquad x \geqq 0. \tag{2}$$

Denote by $\eta(t)$ the number of busy channels at time t and put $\eta_n = \eta(t_n - 0)$. We say that the system is in the state E_k, $k = 0, 1, \cdots, m$ if k channels are busy. Write $P_{k,n} = P(\eta_n = k)$, $k = 0, 1, \cdots, m$, $n = 1, 2, \cdots$, and write $P_k = \lim_{n\to\infty} P_{k,n}$. The limiting distribution $\{P_k\}$ has been obtained by a number of authors, J. W. Cohen [1], C. Palm [2], F. Pollaczek [3], and L. Takács [4]. Introduce the generating function $P_k(w)$, $k = 0, 1, \cdots, m$, defined by

$$P_k(w) = \sum_{n=1}^{\infty} P_{k,n} w^{n-1}, \qquad k = 0, 1, \cdots, m, \ |w| < 1. \tag{3}$$

In this paper we obtain the generating function $P_k(w)$. When $m = \infty$ we obtain the probabilities $P_{k,n}$ explicitly. Our method is a slight generalisation of that of Takács [4]. We remark that in [3] Pollaczek obtained the transient solution in the case $P_{0,1} = 1$ as an application of a very general analytic result.

2. The distribution $\{P_{k,n}\}$**.** We prove the following theorem.

Theorem 1. *Under the assumptions of Section* 1 *we have*

$$P_k(w) = \sum_{r=k}^{m} (-)^{r-k} \binom{r}{k} B_r(w), \qquad |w| < 1, \tag{4}$$

Received February 4, 1960; revised September 12, 1960.

[1] This paper was written under a grant from the Ford Foundation whilst the author was a member of the Research Techniques Division at the London School of Economics.