one finds

$$e^{\frac{1}{2}x_0^2} \frac{\exp\left[-\frac{1}{2}\frac{(x_T - x_0)^2}{2T}\right]}{(2\pi T)^{\frac{1}{2}}} = \int_0^T d\theta (2-\theta)^{\frac{1}{2}} \exp\left[\frac{(x_0+a)^2}{4(2-\theta)}\right] \cdot Q_a(\theta \mid x_0) \frac{\exp\left[-\frac{1}{2}\frac{(x_T - a)^2}{2(T-\theta)}\right]}{[2\pi 2(T-\theta)]^{\frac{1}{2}}}.$$

Integrate on x_T from $-\infty$ to a to obtain

$$\pi^{-\frac{1}{2}} e^{\frac{1}{2}x_0^2} \int_{-\infty}^{(a-x_0)/(2T)^{\frac{1}{2}}} e^{-\frac{1}{2}u^2}\, du = \int_0^T d\theta (2-\theta)^{\frac{1}{2}} \exp\left[\frac{(x_0+a)^2}{4(2-\theta)}\right] Q_a(\theta \mid x_0)\tfrac{1}{2}.$$

Then $Q_a(T \mid x_0)$ can be obtained directly by differentiation with respect to T. A similar derivation can be carried out under the assumption $x_0 < a$. The combined result is

$$Q_a(T \mid x_0) = \frac{|x_0 - a| \exp\left\{-\frac{1}{2}\frac{[x_0(1-T) - a]^2}{T(2-T)}\right\}}{T[2\pi T(2-T)]^{\frac{1}{2}}}, \quad x_0 \neq a, \quad 0 < T \leqq 1.$$

The author has been unable to obtain an expression for $Q_a(T \mid x_0)$ valid for $T > 1$.

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A NOTE ON THE ERGODIC THEOREM OF INFORMATION THEORY[1]

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The purpose of this note is to extend the result of Breiman [1], [2] to an infinite alphabet, or equivalently, the result of Carleson [3] to convergence with probability one.

Let $\{\cdots, x_{-1}, x_0, x_1, \cdots\}$ be a stationary stochastic process taking values in a countable "alphabet" $\{a_i, i = 1, 2, \cdots\}$. Let

$$p(a_{i_1}, \cdots, a_{i_n}) = \mathcal{P}\{x_k = a_{i_k}, k = 1, \cdots, n\},$$

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