

this paper the results of a Monte Carlo analysis of the distribution of  $\delta$  is given. Samples of size  $n = 10, 20, 50, 100,$  and  $500$  were drawn from populations having  $\alpha = -2., -1.1, -1.0, -.9, -.5, 0, .5, .9, 1.0, 1.1$  and  $2.$

**54. Distribution Function for Randomized Factorial Experiments.** S. ZACKS, The Technion, Israel Institute of Technology AND S. EHRENFELD, New York University.

In a previous paper on Randomization and Factorial Experiments [S. Ehrenfeld and S. Zacks, These *Annals*, Vol. 32 (1961), pp. 270-297] two randomization procedures, for choosing fractional replications, were studied. These procedures have been designed to yield information on a subgroup of preassigned parameters. Schemes of the analyses of variance, associated with each of the proposed randomization procedures, were also given. The objective of the present paper is to study the distribution functions of the associated test statistics, and to establish procedures for the determination of test criteria for given levels of significance, as well as the power of the tests.

The distribution functions of the test statistics, for testing the significance of the chosen parameters, depend on the nuisance parameters (those which do not belong to the preassigned subgroup) in a manner that is determined by the randomization procedure. Since the experimenter generally lacks detailed information on the nuisance parameters, the problem is to appraise the sensitivity of the test functions (criteria) to variations in the nuisance parameters.

It is shown that the effect of the nuisance parameters on the distribution function of the test statistics is through statistics of non-centrality, analogous to the parameters of non-centrality of the  $F$ -statistics in the non-randomized case. The low order moments of the statistics of noncentrality are studied, and the distribution functions of the test statistics are approximated by linear contrasts of double non-central  $F$ -distributions multiplied by the central moments of the statistics of non-centrality.

(Abstract not connected with any meeting of the Institute.)

**1. Some Property of a Sequence of Random Events.** MAREK FISZ, University of Warsaw, Poland and University of Washington. (By title)

As the author is aware, the following simple theorem has never been published. Denote by  $A_n (n = 1, 2, \dots)$  a sequence of random events,  $B = \cap A_n$ ,  $p_n = P(A_n)$ ,  $v_n = P(A_{n+1} | \bar{A}_1 \cdots \bar{A}_n)$ . Assume that  $0 < p_n < 1, 0 < v_n < 1 (n = 1, 2, \dots)$ . Then  $P(B) > 0$  if and only if (\*)  $\sum_1^\infty v_n < \infty$ . It is known that if both of the relations  $\sum_1^\infty p_n = \infty$  and (\*\*)  $\sum_1^\infty v_n = \infty$  hold, then  $P(\lim_n \sup A_n) = 1$ . If, however, (\*\*\*)  $\sum_1^\infty p_n < \infty$  and (\*\*) hold, then by virtue of the Borel-Cantelli Lemma,  $P(\lim_n \sup A_n) = 0$  while the probability of occurrence of at least one of the  $A_n$  is positive. If the  $A_n$  are independent, relations (\*) and (\*\*\*) are equivalent and the author's theorem asserts  $P(B) > 0$  while the Borel-Cantelli Lemma asserts the weaker relation  $P(\lim_n \sup A_n) = 0$ .

**CORRECTION TO ABSTRACT**

**"MOMENTS OF THE RADIAL ERROR"**

BY ERNEST M. SCHEUER

The following corrections should be made in the above-titled abstract (*Ann. Math. Stat.*, Vol. 32 (1961), p. 638). Replace sentences two and three by the following:

Let  $\sigma_1^2 = \frac{1}{2}\{(\sigma_{11} + \sigma_{22}) + [(\sigma_{11} - \sigma_{22})^2 + 4\sigma_{12}^2]^{\frac{1}{2}}\}$ ,  $\sigma_2^2 = \frac{1}{2}\{(\sigma_{11} + \sigma_{22}) - [(\sigma_{11} - \sigma_{22})^2 + 4\sigma_{12}^2]^{\frac{1}{2}}\}$ ,  $k^2 = (\sigma_1^2 - \sigma_2^2)/\sigma_1^2$ . Then the moments about the origin  $\mu_n$  of the radial error  $R = [x_1^2 + x_2^2]^{\frac{1}{2}}$  are  $\mu_n' = 2^{\frac{1}{2}}\sigma_1^n \Gamma[\frac{1}{2}(n+2)]F(-\frac{1}{2}n, \frac{1}{2}, 1; k^2)$  where  $F(a, b, c; z)$  is the hypergeometric function.