

## ABSTRACTS OF PAPERS

(Abstracts of papers presented at the Annual Meeting of the Institute, Seattle, Washington, June 14-17, 1961. Additional abstracts appeared in the June, 1961 issue.)

### 18. Tables for the Reliability of Repairable Systems with Time Constraints (Preliminary report). ROQUEZ BEJARANO AND RONALD S. DICK, Inter- national Electric Corp., Paramus, N. J.

Tables have been prepared to solve for the reliability of systems composed of  $A$  similar subsystems of which at most  $N$  can be inoperative for periods exceeding  $t_0$  time units. A second time constraint is introduced into the model so that for at least time  $t_1$  following a return of the system from state  $N + 1$  to  $N$  machines inoperative, the system is only in states 0 to  $N$  or the system fails.

The mixed difference-differential equations solved are of the forms:

$$P_j^1(t) = -[\mu_j + \lambda_j]P_j(t) + \lambda_{j-1}P_{j-1}(t) + \mu_{j+1}P_{j+1}(t) + \mu_{N+1}P_{(N)(j)}(t_1)[P_{N+1}(t - t_1)]$$

$$P_j^2(t) = -[\mu_j + \lambda_j]P_j(t) + \lambda_{j-1}P_{j-1}(t) + \mu_{j+1}P_{j+1}(t) - \lambda_N[P_N(t - t_0)]P_{(N+1)(j)}(t_0).$$

or

$$P_j^{*1}(t) = -\mu_{N+1}P_{(N)(j)}(t_1)[P_{N+1}(t-t_1)] - (\mu_j + \lambda_j)P_j^*(t) + \mu_{j+1}P_{j+1}^*(t) + \lambda_{j-1}P_{j-1}^*(t)$$

where appropriate boundary conditions are applied. Reliability is defined as  $R(t) = \sum_{j=0}^A P_j(t) + \sum_{j=0}^N P_j^*(t)$ . For  $A = 1$  (1) 5, and  $N = 0$ (1) $A - 1$ , the tables give for 81 combinations of  $\lambda$  and  $\mu$  the approximate time at which  $R(t) = .001, .005, .01, .05, .10$  as well as the MTBF. The Cornish-Fisher equation and Weibull approximations are used in finding the reliability points. The MTBF is found by evaluating the Laplace Transforms of the mixed-differential difference equations and is exact. Reference should be made to "The Reliability of Repairable Complex Systems, Part A: The Similar Machine Case" by R. S. Dick, 5th Mil-E-Con National Convention on Military Electronics, 1961 for a complete set of equations solved in this paper and the details of the model.

### 19. Mutual Information and Maximal Correlation as Measures of Dependence. C. B. BELL, San Diego State College.

Kramer (1961) asks if Shannon's mutual information,  $C_P$ , is equivalent to Kramer's generalization (to arbitrary  $\sigma$ -algebras) of Gebelein's (1939) Maximal Korrelation,  $S_P$ , which satisfies Rényi's (1959) postulates for a dependence measure of pairs of random variables. It is found that for two normalizations  $C'_P$  and  $C''_P$  of  $C_P$ : (1)  $0 \leq S_P, C'_P, C''_P \leq 1$ ; (2)  $S_P = 0$  iff  $C_P = 0$  iff  $C_P = 0$  iff the algebras are independent. For strictly positive probability spaces, (3) the algebras are set independent iff there exists a probability function  $P_1$  such that  $S_{P_1} = C_{P_1} = C_{P_1} = 0$ ; (4)  $C'_P = 1$  iff one algebra contains the other; (5)  $C''_P = 1$  iff the algebras are equal; (6)  $S_P = 1$  if the algebras have a non-trivial intersection; (in the finite case, the converse of (6) holds); (7) there exists a probability space such that no two of the dependence measures are equivalent. *Open Problems:* Which of (3)-(6) are valid for (a) the Gelfand-Yaglom (1957) mutual information for non-atomic algebras generated by random variables; and (b) the Lloyd mutual information for arbitrary algebras?