

GENERATING EXPONENTIAL RANDOM VARIABLES

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Introduction. The use of exponential random variables is mentioned in numerous papers on Monte Carlo techniques, in connection with particle or radiation studies, reliability, life testing, etc., but there seems to be little discussion of fast techniques for their generation. Von Neumann discussed a method in [1], where he remarked that in spite of the method's appeal, in that it produced the desired result by performing only discriminations on the relative magnitude of numbers in $(0, 1)$, it was a sad fact of life that it was slightly quicker to use a power series expansion to compute the logarithm of a uniform random variable.

We offer here a simpler device for producing exponential random variables by performing discriminations on the relative magnitudes of uniform $(0, 1)$ random variables. The method is easy to understand, easy to program, requires little storage, and is quite fast, although not quite as fast as one of the versions of the general method given in [2].

The idea is to choose the minimum of a random number of uniform random variables, then add a random integer—roughly, let n and m be random integers taking values according to this schedule:

Value of n	probability	total	Value of m	probability	total
1	.58	.58	0	.63	.63
2	.29	.87	1	.23	.86
3	.10	.97	2	.09	.95
4	.02	.99	3	.03	.98
⋮			⋮		

Then, if u_1, u_2, \dots is a sequence of independent uniform random variables on $(0, 1)$, the random variable

$$(m + \min(u_1, u_2, \dots, u_n))$$

has the exponential distribution. The expected value of n is about 1.58, so that we need on the average only 1.58 u 's from which the minimum must be selected, and 1.58 discriminations to assign a value to n . Assigning a value to m also requires an average of 1.58 discriminations.

Details. We are concerned with a method for expressing an exponentially distributed random variable x in terms of the members of a sequence

$$(1) \quad u_1, u_2, \dots$$

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