ON FIDUCIAL INFERENCE[1]

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1. Introduction. The subject of fiducial probability was introduced thirty years ago by R. A. Fisher. In the original paper [8] entitled "Inverse probability" Fisher discussed the importance of the maximum likelihood method and then produced a *fiducial distribution* for a parameter in roughly the following manner. Let T be a maximum likelihood estimate of a parameter θ. The distribution function for T given θ, $F(T \mid \theta)$, has a uniform distribution on the interval [0, 1]. Differentiating partially with respect to T gives the probability density function for T given θ:

$$\left| \frac{\partial}{\partial T} F(T \mid \theta) \right|$$

Differentiating partially with respect to θ gives a function treated as a density function for "the fiducial distribution of a parameter θ for a given statistic T." From this density function, "fiducial limits" for the parameter θ given T can be calculated.

As an illustration Fisher treated the correlation coefficient r for sampling from a normal bivariate population having correlation ρ. The supporting interpretation for the fiducial method in this example seems to me very much like a present-day confidence argument. This, I gather, led Professor Neyman in 1934 [14] to present his theory of confidence intervals as an *extension* of the fiducial method. Both Fisher and Neyman have since emphasized that the theories are different and the recent literature stands in testimony to the large separation now existing between them.

Today I shall review some of the problems that have been analyzed by the fiducial method and discuss briefly some of the results obtained for these problems; also, I shall put forward a mathematical framework[3] within which I feel fiducial probability has a clear frequency interpretation for a large class of problems. A natural beginning is Fisher's [2] statement: "By contrast, the fiducial argument uses the observations only to change the logical status of the parameter from one in which nothing is known of it, and no probability statement about it can be made, to the status of a random variable having a well defined distribution." Such statements have perturbed many mathematical statisticians.

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[3] The development and the proof will be found in [10].