$P_{st}(y, \cdot) = \hat{P}_{st}(y, \cdot)$ if $y \not\varepsilon M_s$ and $\hat{P}_{rs}(x, M_s) = 0$ for all x it follows that $\hat{P}_{rt} = \hat{P}_{rs} * \hat{P}_{st}$.

REFERENCE

[1] J. L. Doob, *Stochastic Processes*, John Wiley and Sons, New York, 1953.

A GENERALIZATION OF A THEOREM OF BALAKRISHNAN[1]

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1. Introduction. Given a stochastic process $\{X(t), t \varepsilon T\}$ on some probability space with second moment kernel

$$\mathcal{E}[X(s)\overline{X(t)}] = K(s, t),$$

a characterization is given of the function

$$m(t) = \mathcal{E}X(t).$$

This characterization includes the result of Balakrishnan [2] for the case of second order stationary, discrete or continuous parameter processes.

2. The characterization. Let T be an abstract set and let K be a positive definite kernel on $T \times T$. A function m on T is said to be an admissible mean value function for the kernel K if there exists a stochastic process $\{X(t), t \varepsilon T\}$ on some probability space with

$$\mathcal{E}[X(s)\overline{X(t)}] = K(s, t) \quad \text{and} \quad \mathcal{E}X(t) = m(t).$$

Lemma 1. *m is an admissible mean value function for the kernel K if and only if $K(s, t) - m(s)\overline{m(t)}$ is positive definite.*

Proof. if $K(s, t) - m(s)\overline{m(t)}$ is a positive definite kernel on $T \times T$, let $\{X(t), t \varepsilon T\}$ be a Gaussian process with mean function m and covariance kernel $K(s, t) - m(s)\overline{m(t)}$, ([3], p. 72). Then

$$\begin{aligned}\mathcal{E}[X(s)\overline{X(t)}] &= \mathcal{E}[X(s) - m(s)][\overline{X(t) - m(t)}] + m(s)\overline{m(t)} \\ &= K(s, t).\end{aligned}$$

Conversely, if m is admissible,

$$\mathcal{E}[X(s) - m(s)][\overline{X(t) - m(t)}] = K(s, t) - m(s)\overline{m(t)}$$

is positive definite.

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