QUEUES WITH BATCH DEPARTURES I

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1. Introduction. This paper has a pattern closely similar to that of [4]. The following single-server queueing system is considered.

(i) Units arrive at the sequence of instants $\tau_1, \tau_2, \cdots$, such that the inter-arrival times, $\theta_n = \tau_{n+1} - \tau_n > 0$ $(n = 1, 2, \cdots)$, are identically distributed independent random variables with an exponential distribution function,

$$F(x) = P[\theta_n \leqq x] = 1 - e^{-\lambda x} \qquad (x \geqq 0).$$

Put $\alpha = \int_0^\infty x\, dF(x)$. Then $\lambda = 1/\alpha$.

(ii) Units are served in batches of exactly k units by a single server, in order of arrival. Denote by χ_n the service time of the nth batch to be served. We suppose that $\{\chi_n\}$ $(n = 1, 2, \cdots)$ is a sequence of identically distributed independent positive random variables, independent also of the sequence $\{\tau_n\}$, with common distribution function, $H(x) = P[\chi_n \leqq x]$. Put $\psi(s) = \int_0^\infty e^{-sx}\, dH(x)$, $\beta = \int_0^\infty x\, dH(x)$ and $\mu = 1/\beta$. Define $\rho = \lambda/\mu$.

In the terminology of Foster [3], this system can alternatively be described as having the 1-input (arrivals) untriggered with input quantity constantly unity, and an exponential distribution for the 1-input time. The 0-input (departures) is triggered with input quantity constantly, k and a general distribution for the 0-input time. The system has infinite capacity. For definitions of these concepts, the reader is referred to [3].

Such a batch-size model does not appear to have been treated explicitly in the literature, although it has obvious applications. A simple special case of it is, however, implicit in the work of Jackson and Nichols [5]. These authors suppose that an inter-arrival time devoted to one unit is composed of k consecutive phases, each exponentially distributed. If instead, we think of this unit as composed of k subunits (corresponding to the phases of arrival) then we have the idea of batch service: Jackson and Nichols treat the special case of exponential service times.

Justification for the explicit consideration of batch departure systems resides in the fact that the results one can obtain are elegant, and form a natural generalization of the case of unit departures, as treated, for example, in Kendall [6]. The analysis in this paper is similar to that in Bailey [1], but the model is in fact different, and the results obtained here are new. In the terminology of Foster [3], the model Bailey considered differs from the present one in that the 0-input in Bailey's model is untriggered with controlled input quantity of zero to k units, depending upon the state of the system: the input being, for example, virtual when the system contains no 1's. In other words, service begins from time to time

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