LIMIT DISTRIBUTIONS IN THE THEORY OF COUNTERS

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1. Introduction. Let us suppose that particles that arrive on the counter be randomly spaced on the positive time axis. In an actual counter each particle arriving in the time interval $(0, \infty)$ independently of others gives rise to an impulse with probability p or 1 according to whether at this instant there is an impulse present or not. Hence the counter is in one or other of two mutually exclusive states which we denote by A and B. The counter is in state A when no impulse covers the instant and is in state B otherwise and it assumes the states A and B alternatively. A particle striking the counter is recorded if and only if the counter is in state A. If $p = 0$,we get the type I counter and if $p = 1$, we get the type II counter.

Let $t_1, t_2, t_3 \cdots$ be the instants at which particles arrive and $\chi_1, \chi_2, \chi_3, \cdots$ be the lengths of successive impulses. Let $\tau_1, \tau_2, \tau_3 \cdots$ be the instants of successive recordings. Let us assume that the time from an arbitrary point in the positive time axis to the time of arrival of the first particle that follows it is a random variable with distribution function $F(x)$. Hence the differences $\{t_{r+1} - t_r\}$, $r = 1, 2, 3, \cdots$, are identically distributed random variables independent of each other with a distribution function $F(x)$. Let the time durations of the impulses be independent and identically distributed random variables with the distribution function $H(x)$. Let these random variables be independent of the instants of arrivals and of the events of the realizations of the impulses. Let ν_t denote the number of registered particles in the time interval $(0, t)$. Let the process start in state A. Let us denote by $\xi_1, \eta_1, \xi_2, \eta_2, \cdots$ the times spent in states A and B respectively. Consequently $\{\xi_n\}$ and $\{\eta_n\}$ are independent sequences of identically distributed positive random variables. It can be seen that $\Pr[\xi_n \leqq x] = F(x), x > 0$. Let $\Pr[\eta_n \leqq x] = U(x), x > 0$. It can also be seen that the instants of transitions $A \to B$ coincide with the instants τ_n, $n = 1, 2, 3, \cdots$. Hence the time differences $\{\tau_{n+1} - \tau_n\}$ are identically distributed random variables whose distribution function $G(x)$ is given by

$$G(x) = F(x) * U(x) = \int_0^x U(x - y)\, dF(y). \tag{1.1}$$

In [8] Takács has shown that ν_t, suitably normalized, is asymptotically normal. In [9] Takács has considered the asymptotic behavior of ν_t/t. Here he has also applied the law of the iterated logarithm, as stated by P. Hartman and A. Wintner [5], to ν_t. In this paper we consider the asymptotic ($\sim$) behaviour of ν_t when $F(x)$ and $H(x)$ follow the stable laws with suitable characteristic exponents and we show that ν_t, suitably normalized, tends to the Mittag-Leffler distribution for all counters of the types detailed above.

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