

# ON A GEOMETRICAL METHOD OF CONSTRUCTION OF PARTIALLY BALANCED DESIGNS WITH TWO ASSOCIATE CLASSES

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**1. Introduction.** The method of construction of partially balanced block designs discussed here is different than the ones known in the literature. It is based on the existence of an oval (maximum number of points no three on one line) in finite Desarguesian planes. This method can be applied to any plane of order  $2^h$ ,  $h$  a positive integer, i.e., to planes with  $2^h + 1$  points on a line. No general procedure has thus far been obtained for planes with  $p^n + 1$  points on a line,  $p$  an odd prime and  $n$  a positive integer. A design based on a plane with 10 points on a line will be constructed. Further generalizations of this method will be discussed later.

**2. Construction of partially balanced designs based on finite Desarguesian planes with  $2^h + 1$  points on a line.** The total number of points in a plane with  $2^h + 1$  points on a line is  $2^{2h} + 2^h + 1$ . Furthermore it is well known that such planes include ovals consisting of the maximum possible number of points  $2^h + 2$ . The lines of the plane can be classified into two categories with respect to the oval. The first category includes lines having two points of the oval, henceforth called secants. The second category of lines consists of lines not including any point of the oval. The number of lines belonging to each of the two categories is clearly  $(2^{h-1} + 1)(2^h + 1)$  and  $2^{2h-1} - 2^{h-1}$  respectively. Consider now the points of the plane which are not on the oval. Their number is  $2^{2h} - 1$ . Each of them lies on  $2^{h-1} + 1$  secants and  $2^{h-1}$  lines of the second category. This leads to a construction of partially balanced block designs identifying the points with the objects and the lines with the blocks. Each of the two categories of lines taken separately gives rise to a partially balanced block design. The first design will be obtained by calling two objects first associates if the points representing them lie on one secant, second associates otherwise. The second design will be obtained by interchanging the roles of the two categories of lines.

The parameters of the first design are as follows:

$$\begin{aligned} v &= 2^{2h} - 1, & b &= (2^{h-1} + 1)(2^h + 1), \\ r &= 2^{h-1} + 1, & \lambda_1 &= 1, & \lambda_2 &= 0, \\ k &= 2^h - 1, & n_1 &= 2^{2h-1} - 2, & n_2 &= 2^{2h-1}, & n_3 &= 2^{2h-1}, \\ P_1 &= \left( 2^{2h-2} - 3, \begin{matrix} 2^{2h-2} \\ 2^{2h-2} \end{matrix} \right), & P_2 &= \left( 2^{2h-2} - 1, \begin{matrix} 2^{2h-2} - 1 \\ 2^{2h-2} \end{matrix} \right). \end{aligned}$$

The parameters of the second design obtained by identifying the blocks with

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