INFINITELY DIVISIBLE DISTRIBUTIONS: RECENT RESULTS AND APPLICATIONS

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1. Preliminary remarks. The theory of infinitely divisible distributions, developed primarily during the period from 1920 to 1950, has played a fundamental role in the solution of limit problems for sums of independent random variables. A full account of this theory and its applications, as it had been developed by the late 40’s, were presented in the monographs of Lévy [62], Gnedenko and Kolmogorov [29] and Loeve [69]. In the last ten years research in this field has been carried out along many lines. Numerous new results have been obtained and entirely new applications have been found. This paper is an attempt to give a full coverage of the results and applications obtained since 1950. Hence, we consider to be recent, results obtained since the appearance of the Russian edition of Gnedenko and Kolmogorov’s book.

The scope of this paper is basically restricted to one-dimensional, real random variables. A brief mention of multi-dimensional random vectors will be made in Section 3.6, while generalizations of the theory of infinitely divisible distributions to stochastic processes and random elements in abstract spaces will be omitted.

The introduction, Section 2, contains a short presentation of basic concepts and results, to be found in the monographs mentioned above. Recent results and applications are presented, respectively, in Sections 3 and 4.

Throughout the paper the symbols r.v., r.vec., d.f., den.f., ch.f., ind., id.d., i.d., iff., nse., and iwe. will stand, respectively, for “random variable”, “random vector”, “distribution function”, “density function”, “characteristic function”, “independent”, “identically distributed”, “infinitely divisible”, “if and only if”, “necessary and sufficient conditions” and “in the sense of weak convergence”.

The notation used in this paper may differ from that used in the original papers referred to.

2. Introduction. The r.v. $X$, or equivalently its d.f. or its ch.f., will be called i.d. if, for every positive integer $n$, we have $X = Y_{1} \cdots Y_{n}$ with $Y_{k}(k = 1, \cdots, n)$ ind. and id.d. The class of i.d.r.v. will be denoted by $I$.

Khintchine [45a] has shown that the d.f. $F(x)$ is i.d iff. the logarithm of its ch.f. $\varphi(t)$ is representable in the form

$$
\log \varphi(t) = i\gamma t + \int_{-\infty}^{\infty} A(u, t) \frac{1 + u^2}{u^2} \, dG(u)
$$

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