

is minimized by the choice $c = c^*$. A design which minimizes $d(x_0)$ can then be obtained easily from c^* in a manner described in [8]; for our present considerations, we need only mention that $d(x_0) = [m(c^*)]^{-2}$, which can be used to tell us whether or not $x_0 \in B$.

Finally, we remark that the Chebyshev approximation problem just described in terms of the g_i 's can be rewritten as a "modified Chebyshev problem" in terms of the original f_i 's, namely, to minimize

$$\max_x \left| \left[1 + \sum_2^k c_i f_i(x_0) \right] f_1(x) / f_1(x_0) - \sum_2^k c_i f_i(x) \right|.$$

For computational purposes, it is often convenient to solve this problem by first solving the restricted Chebyshev problem of minimizing

$$\max_x \left| f_1(x) / f_1(x_0) - r^{-1} \sum_2^k c_i f_i(x) \right|$$

subject to $\sum_2^k c_i f_i(x_0) = r - 1$, then multiplying the resulting minimum by r and minimizing with respect to r .

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A CONTOUR-INTEGRAL DERIVATION OF THE NON-CENTRAL CHI-SQUARE DISTRIBUTION

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The brief discussion which follows presents a contour-integral derivation of the non-central chi-square distribution. Although this distribution is well

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