in which the s_i are positive with $s_r \geqq s_{r+1}$, $r = 1, \cdots, t - 1$, and $\phi = \min(j, \gamma)$.

Cases in which the a_i are not distinct can be treated as above except that (36) must be replaced by the corresponding limit formulae.

Acknowledgment. The author wishes to thank D. L. Heck for constructive suggestions regarding the preparation of this paper.

REFERENCES

[1] Amoroso, L. (1925). Ricerche intorno alla curva dei redditi. *Ann. Mat. Pura Appl.* Ser. 4 **21** 123–159.

[2] D'Addario, R. (1932). Intorno alla curva dei redditi di Amoroso. *Riv. Italiana Statist. Econ. Fnanza*, anno 4, No. 1.

[3] Edwards, Joseph (1954). *A Treatise on the Integral Calculus*. Chelsea Pub., New York.

[4] Haight, Frank A. (1961). Index to the distributions of mathematical statistics. *J. Research*, National Bureau of Standards, Sec. B **65B** No. 1, 23–60.

[5] Haller, B. (1945). Verteilungsfunktionen und ihre Auszeichnung durch Funktional gleichungen. *Mitt. Verein. Schweiz. Versich.- math.* **45** No. 1, 97–163.

[6] Pearson, Karl (1934). *Tables of the Incomplete Gamma Function*. Cambridge Univ. Press.

[7] Robbins, Herbert (1948). The distribution of a definite quadratic form. *Ann. Math. Statist.* **19** 266–270.

IMPROVED BOUNDS ON A MEASURE OF SKEWNESS

By Kulendra N. Majindar

Delhi University, India

In 1932, Hotelling and Solomons [2] proved that the absolute value of a certain measure of skewness for a population can not exceed 1. This result has been used by Madow [3] in his study of systematic sampling. The proof given by Hotelling and Solomons covers the case of a discrete random variable. In this note we extend and strengthen the inequality for any random variable with a positive standard deviation. Let X be a random variable with a positive standard deviation, M its median and $F(x)$ its cumulative distribution function. If the median is not uniquely defined, we will define it by $M = \frac{1}{2}\sup\{x : F(x) < \frac{1}{2}\} + \frac{1}{2}\inf\{x : F(x) > \frac{1}{2}\}$. The measure of skewness, S, considered here is the ratio of the difference between the mean and median to the standard deviation of X. With this definition we establish the following theorem.

Theorem. *The measure of skewness S of a random variable X with a finite positive standard deviation satisfies the inequality*

$$|S| < 2(pq)^{\frac{1}{2}}/(p + q)^{\frac{1}{2}},$$

where $p = \Pr\{X > E(X)\}$ and $q = \Pr\{X < E(X)\}$.

Received June 26, 1961; revised February 26, 1962.