ON LINEAR ESTIMATION FOR REGRESSION PROBLEMS ON TIME SERIES[1]

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1. Introduction. The purpose of this paper is to discuss, in a general context, certain mean value problems of a single parameter nature. More specifically, suppose one observes the family $\{Y(t),\ t \varepsilon T\}$ of random variables satisfying $Y(t) = m(t, \beta) + X(t), t \varepsilon T$, where β is an unknown parameter lying in some set Λ. Suppose further that

$$E_\beta X(t) \equiv 0, \qquad t \varepsilon T, \qquad \beta \varepsilon \Lambda,$$

$$E_\beta X(s)X(t) = K(s, t), \qquad s, t \varepsilon T, \qquad \beta \varepsilon \Lambda.$$

(Assumptions will be imposed on the set T and the kernel K where necessary.) The problem of interest is that of estimating, on the basis of the observation, a real valued function g defined on Λ under the criterion of squared error loss. This model emanates from regression analysis on time series and our attention is focussed in this direction. The estimators concerned will be linear only and we shall be interested in problems for which the mean value function is not essentially linear in β.

The results obtained proceed from the application of results from the area of reproducing kernel spaces. The statistical results include a specification of problems in which linear estimation makes sense, a precise lower bound for the risk function of a linear estimator, and a characterization of those problems which admit consistent (zero risk) linear estimators.

As previously mentioned, the tools arise in the theory of reproducing kernel spaces and consequently we begin, in Section 2, by listing the appropriate results of this discipline. Section 3 is devoted to the properties of linear estimators, and Section 4 is given over to examples and remarks.

As few assumptions are imposed as is feasible and specific problems are introduced only for purposes of illustration.

This work leans heavily, for its emphasis, on the work of Parzen, [4] and [5], in this same vein, as will frequently be noted in the sequel.

2. Reproducing kernel spaces. The theory of reproducing kernel spaces has received an extensive exposition in the paper of Aronszajn [1]. The purpose of this section will be to state succinctly the apparatus necessary to accomplish the next section. For a much broader discussion of the role of this theory in the area of time series analysis, the interested reader can consult [4] and [5].

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