EXACT OPERATING CHARACTERISTIC FOR TRUNCATED SEQUENTIAL LIFE TESTS IN THE EXPONENTIAL CASE

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1. Introduction. Non-truncated sequential life tests involving the exponential distribution, $f(t) = \exp(-t/\theta)/\theta$, $t \geqq 0$, $\theta \geqq 0$, have been treated extensively by Epstein and Sobel. In [3], the details are given for

(i) computing the approximate OC curve associated with the familiar sequential probability ratio (A, B) rule, where $A = (1 - \beta)/\alpha$, $B = \beta/(1 - \alpha)$, and where α and β are the nominal errors of first and second kind, respectively;

(ii) computing the exact strength (α', β') of the (A, B) rule where $\alpha' \leqq \alpha$, $\beta \leqq \beta' \leqq \beta/(1 - \alpha)$; and

(iii) determining an (A^*, B) rule with strength exactly (α, β) (based on the solution by Dvoretzky, Kiefer, and Wolfowitz, [1], for the exact OC curve for the non-truncated case).

The above results constitute essentially a complete solution for the OC curve in the non-truncated case. Many times, however, it is desirable to truncate the sequential test after some pre-selected V_0 time units or i_0 failures have been observed. Then if a decision has not been reached earlier, accept H_0: $\theta = \theta_0$ if V_0 is observed before i_0 failures, otherwise accept H_1: $\theta = \theta_1$. Upper bounds on the strength of truncated tests have been given by Epstein in [2].

The main purpose of this paper is to determine the exact OC curve for the truncated test. This result is conveniently obtained as the sum of a finite series whose terms are defined recursively (in Sections 3 and 4) by modifying the Dvoretzky, Kiefer, and Wolfowitz solution [1] for the non-truncated test.

Only the case of sequential testing with replacement is considered. For convenience it is assumed that the sample units are tested one at a time. Extension to the case of testing n units simultaneously (with replacement of failures as they occur) is straightforward (Epstein [2]).

2. Preliminaries and notation. Application of Wald's sequential probability ratio test [3, 5] to the exponential distribution yields the following (A, B) rule:

$$\text{Accept } H_0: \theta = \theta_0 \quad \text{if} \quad V(t) \geqq a_i ,$$

$$\text{Accept } H_1: \theta = \theta_1 \quad \text{if} \quad V(t) \leqq r_i ,$$

Continue test otherwise,

where

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