TESTING APPROXIMATE HYPOTHESES IN THE COMPOSITE CASE[1]

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1. Introduction. One of the chief reasons for not using the Kolmogorov-Smirnov tests is that, as originally presented, they were only suitable for testing the simple hypothesis $F = F_0$ against all alternatives. In a previous paper [3], the author investigated these tests, and extended them to eliminate the paradox of almost sure rejection of the null hypothesis when too much data is observed. Also Kac, Kiefer and Wolfowitz in [1], investigated extensions of the Kolmogorov-Smirnov tests for testing "larger" null hypotheses by means of minimum distance methods. Mention was made in [1] of the difficulty of computing the test statistic. Due to this latter difficulty they suggested a test of normality in which the composite null hypothesis is essentially reduced to a simple one by replacing μ and σ^2 by their estimates $\bar{X}_n$ and s_n^2. Such a test suffers from the disadvantage that distributions which are distance-wise "close" to being normal can lead to rejection of the hypothesis of normality with high probability (since closeness of distribution does not imply closeness of their corresponding moments).

In this paper the basic theory for such tests is briefly developed, and then attention is turned to the practical problem of performing the tests, with round-off error taken into account. Two classes of tests of translation-scale parameter families are presented. They can be performed in a finite number of operations, and have the property that distributions in a "neighborhood" of some member of the family will lead to acceptance of the null hypothesis with at least a specified probability, while distributions at least a specified distance from all such neighborhoods will lead to rejection of the null hypothesis with at least a given probability. Though not done explicitly in this paper, it is clear that the methods developed could be extended to n-parameter families in certain cases.

Essentially, this paper is an extension of the work originated in [3] (which did for the Kolmogorov-Smirnov tests what the paper [2], of Hodges and Lehmann did for the chi-square test), to cases of richer null hypotheses.

2. Some theory of testing hypotheses based on the use of a metric. Assume $X_1, \cdots, X_n$ to be independent random variables with a common distribution function, and let F_n be the random process whose value is the empirical distribution function formed from the observed values of $X_1, \cdots, X_n$. Let $\mathfrak{D}$ be

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