ZERO CROSSING PROBABILITIES FOR GAUSSIAN STATIONARY PROCESSES[1]

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1. Introduction. Let $X(t)$ be a separable stationary Gaussian process with mean zero, $EX(t) \equiv 0$, and continuous covariance function

$$\rho(t) = EX(\tau)X(\tau + t)$$

normalized so that $\rho(0) = 1$. Questions relating to the probability

$$H_X(T) = P[X(t) > 0, 0 \leqq t \leqq T] \tag{1}$$

that $X(t)$ does not cross zero during some time interval T arise in various applications [5], [6].

Many of the difficulties that arise in treating such questions are due to the fact that most of the interesting stationary Gaussian processes are not Markovian. Here we shall obtain bounds on the behavior of $H_X(T)$ particularly for large T using an interesting inequality of D. Slepian [6] and estimates of $H_X(T)$ for some simple processes.

Slepian's inequality states the following: if $X(t)$ and $Y(t)$, $0 \leqq t \leqq T$, are two separable Gaussian processes with mean zero and covariance functions $\rho(t, \tau)$ and $r(t, \tau)$ respectively, normalized so that $\rho(t, t) = r(t, t) = 1, 0 \leqq t \leqq T$, and if

$$\rho(t, \tau) \leqq r(t, \tau), \qquad 0 \leqq t, \tau \leqq T, \tag{2}$$

then

$$H_X(T) \leqq H_Y(T). \tag{3}$$

The inequality is true for either continuous or discrete parameter processes.

2. An upper bound.

Theorem 1. *If $X(t)$ is a separable stationary Gaussian process with $EX(t) \equiv 0$ and $\rho(t) \to 0$ for $t \to \infty$, then*

$$H_X(T) = o(T^{-\alpha}) \tag{4}$$

as $T \to \infty$ for every $\alpha > 0$.

To prove this we first note that

$$H_X(T) \leqq P[X(j\Delta) > 0, j = 0, 1, \cdots, n - 1] \tag{5}$$

for any $\Delta > 0$, $n = [T/\Delta]$ where $[x]$ is the greatest integer less than or equal to x.

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