zero entries in $m \varepsilon M$ determines whether or not $m \varepsilon I$. Now $A'(1), A'(2), \cdots$ is an increasing sequence of subsets of M', which has less than 2^{n^2} elements, so there must be a smallest r, $1 \leqq r \leqq 2^{n^2}$, such that $A'(r) = A'(r + 1)$. To complete the proof we need only show that $A'(r) = A'$ since if $A(r) \subset A(2^{n^2}) \subset I$ then $A' = A'(r) \subset I'$ so $A \subset I$. Thus we need only prove that if $k \geqq 1$ and $A'(k) = A'(k + 1)$ then $A'(k + 1) = A'(k + 2)$. Now if $m \varepsilon A(k + 2)$ then $m = bc$, where $b \varepsilon A(k + 1)$ and $c \varepsilon A(1)$ so there exists a $d \varepsilon A(k)$ with $b' = d'$ so $m' = (dc)' \varepsilon A'(k + 1)$ and the proof is complete.

We conclude with three comments. Clearly $A'(1)$ determines whether or not $A \subset I$ so that if $A(1)$ is an infinite set, which is not the case for indecomposable channels, then $A(1)$ may, for the purpose of determining whether or not $A \subset I$, be replaced by any finite $B \subset M$ with $B' = A'(1)$. If $m \varepsilon A$ has a state which is periodic with period $d > 1$ then $m^d \not\varepsilon I$ and $m^d \varepsilon A$ so $A \not\subset I$. For any $A(1)$, $(A(2^{n^2}))' = A'$.

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NOTE ON QUEUES IN TANDEM[1]

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1. Introduction. Assume that Q_k, $k = 1, 2, \cdots, m$, is a single server queue where customers are served with an exponential service time distribution of mean $1/\mu_k$. We shall assume that the jth customer, C_j, arrives at Q_1 at time t_j, where $\{t_j\}$ are the events of a Poisson process, and λ the number of arrivals per unit time. The queues Q_k are arranged in tandem; that is, after C_j's service at Q_k is completed he proceeds to Q_{k+1} and joins the queue there. We shall extend a result of our previous paper [1] for the foregoing situation.

Let T_{jk} denote C_j's waiting time at Q_k, *including* the duration of C_j's service at Q_k. The purpose of the present note is to show, using the results of [1], that under "equilibrium" conditions the probabilistic description of the random

Received March 26, 1962.

[1] This work was done with support under Contract Nonr-710(16).