ON QUEUES IN TANDEM[1]

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1. Introduction and summary. It was in 1952 that D. V. Lindley [1] obtained the steady-state distribution function of the waiting-time in a single-server queue for the case when the interarrival times are independent random variables with identical probability distributions having a finite mean. He applied the same restrictions to the service times. The resulting waiting time distribution was shown to be the solution of an equation of the Wiener-Hopf type.

Queues in tandem have only recently been studied. In 1957, E. Reich [2] found that in "equilibrium" whereas a non-saturating exponential distribution of interarrival times together with an exponential distribution of service times yields a stationary exponential distribution of interdeparture times, "no such simple behaviour can be expected when the service time distributions are even slightly more general." More recently, J. Sacks [3] has found criteria similar to Lindley's for the existence of steady-state distributions of waiting-times in a finite number of single-server queues in tandem.

The motivation for the work reported in this paper originated in a talk given on April 15, 1958, before the Operations Research Seminar of the University of Michigan by G. D. Camp, who made the following intuitive assertion. "Suppose that we imagine an infinite number of identical servers connected in series, and inject any non-saturating input into the first one. Then we expect the statistics of the outputs to change progressively from server to server and since we are dealing with a diffusion process, it seems intuitively obvious that some equilibrium statistics will be approached (the proof is here left to professional mathematicians)." Also, in a talk given on October 16, 1958, before the Institute of Management Science in Philadelphia, he asserted that in this same queueing system, the probability that the time between the ith and the $i + 1$st customers from the nth service point is less than x approaches, as $n \to \infty$, a probability distribution function $F(x)$, i.e. $F(x)$ is monotone increasing, $F(+\infty) = 1$ and $F(-\infty) = 0$.

It is shown below that these assertions are not true, at least as far as the interdeparture time of the first and second customers is concerned. However, in the unique case of constant service time, the assertions are true and statistical equilibrium is achieved by the output from the first server.

2. Glossary of terms and symbols. *Customer*—An object, animate or inanimate, which enters a queueing system requiring service. *Service*—An operation

Received June 6, 1960; revised May 26, 1962.

[1] Revised with support (S. S.) by NSF G 14093.

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