THE ESTIMATION OF A FUNDAMENTAL INTERACTION PARAMETER IN AN EMIGRATION-IMMIGRATION PROCESS

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0. Synopsis. A convenient method for estimating the infinitesimal transition parameters of a continuous time, multivariate, Markovian emigration-immigration process $\{\mathbf{n}(t)\}$ is described when these can be expressed as known functions of a single fundamental interaction (or migration) parameter θ. The estimator for θ is constructed from the observed generalized mean square consecutive fluctuation in a realised sequence of $\{\mathbf{n}(t)\}$ at a finite number, k, of points in time: $t = 0, \tau, 2\tau, \cdots, (k-1)\tau$. Formulae are derived for the large-sample variance (k large) of the estimator, and its relative efficiency in such samples is investigated.

1. Introduction and summary. Consider the multivariate emigration-immigration or Poisson-Markov process $\{\mathbf{n}(t)\}$ in continuous time with state space the set of all vectors in Euclidean m-space having non-negative integral components. Some properties of this process were derived in a previous paper (Ruben, 1962) with special emphasis on the joint distribution of $\mathbf{n}(t_1), \mathbf{n}(t_2), \cdots, \mathbf{n}(t_k)$ from the point of view of generating functions and on the moments and product moments of the process. We recall in particular that the process is strictly stationary and Markovian and is completely described in terms of $\boldsymbol{\nu}$ and $\boldsymbol{\Lambda}$, where $\boldsymbol{\nu}$ is the process mean, $\boldsymbol{\nu} = E\mathbf{n}(t)$, and $\boldsymbol{\Lambda}$ is an interaction-rate matrix defined in terms of infinitesimal transition parameters λ_{rs}, λ_r^* $(r, s = 1, 2, \cdots, m; r \neq s)$. Further, the covariance matrix of the process is $\mathbf{NP}(t)$ for $t \geqq 0$, where $\mathbf{N}$ is the diagonal matrix with diagonal elements $\nu_1, \cdots, \nu_m$ (the components of $\boldsymbol{\nu}$) and[2]

$$\mathbf{P}(t) = \boldsymbol{\Theta}^{-1}\mathbf{K}(t)\boldsymbol{\Theta}. \tag{1.1}$$

Here $\boldsymbol{\Theta}$ is the matrix of row eigenvectors of $\boldsymbol{\Lambda}$ ($\boldsymbol{\Theta}$ reduces $\boldsymbol{\Lambda}$ to diagonal canonical form) and $\mathbf{K}$ is the diagonal matrix with diagonal elements $\exp(-\kappa_1 t), \cdots, \exp(-\kappa_m t)$, κ_r denoting the (real positive) eigenvalues of $\boldsymbol{\Lambda}$.

The purpose of the present paper is to derive a convenient method of estimation for the infinitesimal transition parameters, and therefore for the transition probabilities,[3] of $\{\mathbf{n}(t)\}$ when these can be related in a known manner to a single unknown fluctuation parameter θ, that is, when the given stochastic process is a member of a singly-infinite class of vector Poisson-Markov processes.

Received March 21, 1961; revised July 16, 1962.

[1] Research sponsored in part under Contract AF 41(657)-214, Teachers College, Columbia University, 1959–60.

[2] Note incidentally that (1.1) implies $\mathbf{P}(t) = \exp(-\boldsymbol{\Lambda}t)$ for $t \geqq 0$.

[3] Any estimate of θ is, of course of predictive value. Prediction of $\mathbf{n}(t+\tau)$ from $\mathbf{n}(t)$ is here facilitated by the property of linear regression of $\mathbf{n}(t+\tau)$ on $\mathbf{n}(t)$ (Ruben, 1962).