

A DEFINITION OF SUBJECTIVE PROBABILITY¹

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1. Introduction. It is widely recognized that the word "probability" has two very different main senses. In its original meaning, which is still the popular meaning, the word is roughly synonymous with plausibility. It has reference to reasonableness of belief or expectation. If "logic" is interpreted in a broad sense, then this kind of probability belongs to logic. In its other meaning, which is that usually attributed to it by statisticians, the word has reference to a type of physical phenomena, known as random or chance phenomena. If "physics" is interpreted in a broad sense, then this kind of probability belongs to physics. Physical probabilities can be determined empirically by noting the proportion of successes in some trials. (The determination is inexact and unsure, like all other physical determinations.)

In order to distinguish these two main senses, physical probabilities will be referred to as "chances," whereas "probability" unqualified will refer to logical probability.

Within the two main categories of logical probability (probability proper) and physical probability (chances), especially in the former, various lesser differences of meaning can be distinguished. In this paper we are concerned with the personal or subjective concept of probability, as considered by Ramsey [13] and Savage [14]. Probabilities and utilities are defined in terms of a person's preferences, in so far as these preferences satisfy certain consistency assumptions. The definition is constructive; that is, the probabilities and utilities can be calculated from observed preferences.

Some persons, especially those with scientific training, are acquainted with the mathematical theory of chances and consider it to be an adequate theory for some kinds of physical phenomena—the uncertain outcomes of the spin of a roulette wheel, the toss of a coin, the roll of a die, a random-number generator. They believe that equipment can be found whose output conforms well with the theory of chances, with stochastic independence between successive observations, and *with stated values* for the chances of the simple outcomes. It suffices for this purpose that they believe that some one such piece of equipment exists, for example, a fair coin, since any system of chances can be realized by multiple use of such equipment. (The relation of the theory of chances with chance phenomena has been well illustrated by Kerrich [6]. See also Neyman [11].)

For such a person, his utilities can be defined in terms of chances, as shown by von Neumann and Morgenstern [10]. The purpose of this note is to define

Received August 14, 1961; revised August 16, 1962.

¹ Research partly supported by the Office of Naval Research, in the Department of Mathematics and in the Econometric Research Program, Princeton University.