SOME RESULTS ON THE DISTRIBUTION OF TWO RANDOM MATRICES USED IN CLASSIFICATION PROCEDURES

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1. Introduction and summary. Wald [5] and Anderson [1] discuss a classification problem as follows. We have $N_1 + N_2 + 1$ independent p dimensional random vectors. We know that the first N_1 vectors are observations from a population Π_1, the following N_2 are observations from a population Π_2, and the last vector is either from Π_1 or Π_2. Let us assume that the probability distribution in both Π_1 and Π_2 is multivariate normal with the same covariance matrix Σ; the vector of expected values being μ_1 in Π_1 and μ_2 in Π_2. The values of μ_1, μ_2, and Σ are unknown. Let X denote the $p\cdot(N_1 + N_2 + 1)$ matrix of observations. On the basis of X we wish to classify the last observation, $X_{N_1+N_2+1}$ as coming from Π_1 or Π_2. For this purpose both Wald and Anderson propose classification statistics. Wald proposes the statistic

$$U = X'_{N_1+N_2+1}S^{-1}(\bar{X}_1 - \bar{X}_2), \tag{1.1}$$

where

$$\bar{X}_1 = (1/N_1)\sum_{t=1}^{N_1} X_t, \qquad \bar{X}_2 = (1/N_2)\sum_{t=N_1+1}^{N_1+N_2} X_t, \tag{1.2}$$

and

$$\begin{aligned} S &= (1/N_1 + N_2 - 2) \\ &\cdot\left[\sum_{t=1}^{N_1}(X_t - \bar{X}_1)(X_t - \bar{X}_1)' + \sum_{t=N_1+1}^{N_1+N_2}(X_t - \bar{X}_2)(X_t - \bar{X}_2)'\right]. \end{aligned} \tag{1.3}$$

Anderson considers the statistic

$$W = X'_{N_1+N_2+1}S^{-1}(\bar{X}_1 - \bar{X}_2) - \tfrac{1}{2}(\bar{X}_1 + \bar{X}_2)'S^{-1}(\bar{X}_1 - \bar{X}_2). \tag{1.4}$$

It is known [4] that the sampling distributions of U and W are contained as special cases in the sampling distribution of the statistic

$$V = Y_1'A^{-1}Y_2, \tag{1.5}$$

where Y_1 and Y_2 are p dimensional random normal vectors with mean vectors ξ_1 and ξ_2, say, respectively, and A is a $p \times p$ symmetric matrix having the Wishart distribution with n degrees of freedom; the three sets are independently distributed with the same covariance matrix Σ. Let M denote the matrix

$$M = \begin{pmatrix} m_{11} & m_{12} \\ m_{12} & m_{22} \end{pmatrix} = Y'B^{-1}Y, \tag{1.6}$$

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