STATISTICAL ANALYSIS BASED ON A CERTAIN MULTIVARIATE COMPLEX GAUSSIAN DISTRIBUTION (AN INTRODUCTION)[1]

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0. Summary. A complex Gaussian random variable is a complex random variable whose real and imaginary parts are bivariate Gaussian distributed. A p-variate complex Gaussian random variable is a p-tuple of complex Gaussian random variables such that the vector of real and imaginary parts is $2p$-variate Gaussian distributed. The present paper is an introduction to statistical analysis based on a certain multivariate complex Gaussian distribution which is the distribution of a p-tuple of complex Gaussian random variables whose real and imaginary parts are $2p$-variate Gaussian distributed with a $2p \times 2p$ real covariance matrix of special form. The special form of the $2p \times 2p$ real covariance matrix permits the distribution of the p-tuple of complex Gaussian random variables to be expressed in complex form, and to be (in the zero mean case) specified by a $p \times p$ Hermitian covariance matrix. (Certain simplifying conditions, e.g. zero mean random variables, non-singularity of matrices, etc. are retained throughout the paper. Such conditions may be removed at the expense of added complexity of exposition or results.) Statistical analysis based on the particular multivariate complex Gaussian distribution mentioned above possesses certain desirable properties:

(1) A theory that is a counterpart of (i.e. "parallels") classical multivariate real Gaussian statistical analysis may be developed.

(2) From the methods of proof and the distributional results stated in the paper there are indications that for every distributional result of classical multivariate real Gaussian statistical analysis obtainable in closed (explicit) form, the counterpart result in the multivariate complex Gaussian statistical analysis is also obtainable in closed (explicit) form. A comparison between certain counterpart distributional results of multivariate complex and real Gaussian statistical analysis indicates that the multivariate complex Gaussian distributional results often appear formally simpler and at times are simpler. Furthermore, not all distributional results of the multivariate complex Gaussian statistical analysis are counterpart results of multivariate real Gaussian statistical analysis so that,

Received April 4, 1960; revised August 16, 1962.

[1] The research summarized in the paper was supported (in part) by the American Cyanamid Company, New York City and the Lincoln Laboratories of M.I.T. on a contract to the Mathematics Department, College of Engineering, New York University. Part of the research was carried out while the author was a faculty member of New York University and a consultant for the American Cyanamid Company. Additional support for preparation of the revised manuscript was obtained from the Rocketdyne Division of North American Aviation, Inc.

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